Wigner's Friend as a Gluing Problem: Category Theory, Homotopy Type Theory, and the Relational Structure of Quantum Reality

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Wigner's friend exposes a deep fracture in the classical assumption that reality must be representable as a single observer-free inventory of facts. If the friend inside a sealed laboratory records a definite outcome while Wigner outside consistently assigns a superposed state to the entire lab, then the quantum state cannot be treated simply as a universal container of completed truths. This paper reframes the problem through category theory and homotopy type theory, where observers become contexts, measurements become morphisms, records become typed witnesses, and later communication becomes a gluing operation over shared refinements. The central proposal is that quantum reality is not a raw state vector but a coherent diagram of equivalences among state assignments. Global classical facts appear only when local perspectives become mutually compatible through interaction. In this view, Wigner's friend is not a paradox to be dissolved by choosing one observer as privileged, but a categorical obstruction to naïve global truth. The result is an equivalence-first account of quantum foundations grounded in context, transport, univalence, and relational consistency.

Keywords: Wigner's friend, quantum foundations, category theory, homotopy type theory, HoTT, relational quantum mechanics, quantum contextuality, quantum measurement, univalence, sheaf theory, observer-relative facts, quantum state, global sections, categorical physics, equivalence-first physics.

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1. Introduction: From Quantum States to Contextual Reality

The Wigner's friend thought experiment remains one of the most compact and unsettling probes of quantum foundations because it forces a distinction between a quantum state as a formal description and reality as a structure of physically meaningful relations. In the ordinary presentation, a friend inside a sealed laboratory measures a quantum system and obtains a definite result. From the friend's perspective, a fact has occurred. A record exists. Memory, apparatus, notebook, and environment have become aligned around one outcome. Yet Wigner, standing outside the closed laboratory, may consistently describe the entire lab, including the friend, apparatus, and recorded outcome, as a quantum superposition. The problem is not simply that two observers disagree. Disagreement is common in classical physics when observers possess different information. The deeper problem is that both descriptions can appear licensed by quantum theory itself. The friend has a definite measurement record; Wigner has a unitary description of the unopened lab. If quantum states are supposed to describe what is real, then the same physical situation seems to support two incompatible claims about reality.

This paper begins from the premise that the contradiction is not solved by asking which observer is psychologically mistaken. The real issue is structural. Quantum theory does not behave like a classical registry in which all facts already exist as entries in a universal database, waiting to be consulted. Instead, the theory appears to assign different state descriptions relative to different physical contexts, interaction histories, and available records. Wigner's friend is therefore not merely a puzzle about consciousness, knowledge, or laboratory isolation. It is a test case for whether physical reality should be understood as a single absolute state or as a coherent diagram of context-dependent state assignments whose mutual consistency is enforced only through interaction.

1.1. The Problem of the Universal State

The phrase "the quantum state of the universe" suggests a complete and final description. It tempts us to imagine a single object, perhaps an immense vector in a universal Hilbert space, containing every possible fact about every subsystem, observer, record, and event. On this view, the quantum state is the deepest inventory of reality. Everything else is derivative. Measurement outcomes, observers, laboratories, memories, histories, and classical objects are all supposed to arise from this universal state.

But Wigner's friend makes this picture unstable. If the friend has measured the system and recorded an outcome, then the friend's local world contains a definite fact. Yet if Wigner has not interacted with the laboratory, he may treat the combined system as evolving unitarily. The universal-state picture now faces an uncomfortable choice. Either the friend's fact is not yet

fully real from the outside, or Wigner's superposition is merely a statement of ignorance, or there are multiple branches, or state assignment itself is not absolute. Each option reveals that the phrase "the state of reality" is less transparent than it first appears.

The universal-state problem is not that quantum theory lacks mathematical precision. The problem is that precision in formal description does not automatically imply a single classical interpretation. A state vector may encode amplitudes, correlations, symmetries, phases, and transition rules, but it does not automatically encode one observer-independent table of all definite outcomes. In ordinary classical reasoning, one assumes that every well-formed question has, at least in principle, a definite answer. Quantum mechanics challenges that assumption. It does not merely hide answers from observers; it restricts what counts as a jointly meaningful set of answers.

For this reason, the universal state should not be treated naively as a cosmic database. It may be better understood as a highly structured object whose physical content is extracted only through admissible relations: measurement, interaction, transformation, equivalence, and communication. The question then becomes not "What facts are already written into the universal state?" but "Which facts are stable under the relevant physical maps?"

1.2. Why Wigner's Friend Remains Foundational

Wigner's friend remains foundational because it dramatizes the boundary between private record and public consistency. A measurement inside the lab is not an abstract event. It produces a record for the friend. The friend can remember the outcome, write it down, communicate it later, and use it to make predictions. For the friend, the outcome is not merely possible. It is actual. Yet Wigner's external description need not select that same actual outcome until Wigner interacts with the lab.

This is why the thought experiment cannot be dismissed as a minor oddity about observers. It asks what kind of thing a quantum state is. Is it a physical object, an epistemic bookkeeping device, a branch-relative description, an agent-relative probability assignment, a relational structure, or something else? Any interpretation of quantum mechanics must answer this question, even if it answers indirectly.

The scenario is especially important because it brings the observer into the domain of the theory. In simpler measurement discussions, one may imagine a microscopic system on one side and a classical measuring device on the other. Wigner's friend removes that comfortable division. The measuring apparatus, the friend's body, the friend's memory, and the written record are all physical systems. If quantum mechanics applies universally, then they too must be describable quantum mechanically. But if they are described quantum mechanically, the

apparent definiteness of measurement becomes difficult to reconcile with an outside unitary description.

The foundational power of Wigner's friend lies in this forced symmetry. The friend is not outside physics. Wigner is not outside physics. The lab is not outside physics. No observer can be granted automatic metaphysical privilege merely because that observer is the one telling the story. The theory must explain how definite facts arise locally, how they become shared globally, and why different state assignments do not produce outright contradiction when observers eventually interact.

1.3. From Observer-Independent Facts to Context-Indexed Judgments

A more precise way to formulate the issue is to say that quantum facts are context-indexed judgments. The friend's statement "I saw outcome 0" is not merely a timeless proposition floating free of circumstance. It is a judgment made within a physical context: the friend has interacted with the system, the apparatus has registered a value, the memory has been updated, and the local laboratory environment has become correlated with that record. The fact is real in that context because it is supported by a chain of physical relations.

Wigner's statement is different. Before opening the lab, Wigner may assign a superposed state to the entire friend-laboratory system. This is also a context-indexed judgment. It belongs to Wigner's external standpoint, in which no interaction has yet occurred that would select a definite friend-record for Wigner. The friend's fact and Wigner's state assignment appear contradictory only if the context labels are erased.

This shift from observer-independent facts to context-indexed judgments does not mean that reality becomes arbitrary, subjective, or unconstrained. Contexts are not fantasies. They are physical situations defined by interaction histories and accessible records. A context-indexed fact is not whatever an observer happens to believe. It is a claim stabilized by the relations available within that context. When contexts later interact, their records must satisfy compatibility conditions. The friend cannot tell Wigner one result while the laboratory record, apparatus state, and later correlations consistently indicate another without some further physical explanation.

Thus contextuality is not chaos. It is disciplined locality of fact. Facts become definite within contexts, and broader objectivity emerges when contexts are glued together through communication, comparison, and shared records. Classical reality is not assumed at the start; it is recovered as a stable pattern of compatible relations.

1.4. The Paper's Equivalence-First Thesis

The central thesis of this paper is that Wigner's friend should be understood as a gluing problem in an equivalence-first ontology. Reality is not the raw quantum state vector. Reality is the coherent structure of relations among state assignments, records, observers, and transformations. A state vector by itself may contain representational redundancy, inaccessible amplitude structure, basis dependence, global phase freedom, and gauge-like artifacts. What becomes physically real is what survives the appropriate equivalences and can be transported consistently across contexts.

Category theory and homotopy type theory provide natural languages for this thesis. Category theory allows observers, laboratories, systems, and measurement situations to be treated as objects in a structured network, while interactions, measurements, restrictions, and communications appear as morphisms. Homotopy type theory allows facts to be treated as typed judgments whose validity depends on context and whose identity is governed by equivalence. Univalence then expresses a powerful foundational idea: descriptions that are equivalent in the relevant structure-preserving sense may be treated as the same physical content.

In this framework, the friend's definite record and Wigner's external superposition are not two isolated metaphysical claims competing for absolute authority. They are local descriptions within different contexts. The problem is whether these descriptions can be related, refined, transported, and ultimately glued into a coherent shared structure when interaction occurs. If they can, then the apparent contradiction is not a contradiction in reality but a failure of classical language to track contextual domains. If they cannot, then the obstruction reveals something genuinely nonclassical about the structure of facts.

The equivalence-first thesis can therefore be stated plainly: quantum reality is not a universal list of completed facts encoded in one privileged state; it is a coherent diagram of state assignments whose physical meaning lies in the equivalences, morphisms, records, and compatibility conditions connecting them. Wigner's friend matters because it shows that the deepest object of quantum foundations may not be the state itself, but the structured relation among perspectives from which stable reality is assembled.

2. The Wigner's Friend Scenario as a Structural Problem

The Wigner's friend scenario is often introduced as a paradox about observation, but its deeper significance lies in the structure of description itself. The thought experiment does not merely ask whether consciousness collapses the wavefunction or whether one observer knows something another does not. It asks whether quantum theory permits a single, observer-

independent account of all facts when observers themselves are treated as quantum systems. Once the friend, the measuring apparatus, the record, and the laboratory are included inside the physical description, the classical separation between observer and observed begins to fail. The observer is no longer a metaphysical outsider who reads reality from beyond the system. The observer becomes part of the same network of interactions that the theory must describe.

At the heart of the scenario is a mismatch between two valid-seeming descriptions. Inside the laboratory, the friend performs a measurement and obtains a definite result. Outside the laboratory, Wigner may assign a superposed state to the entire sealed lab. The friend describes an event that has happened. Wigner describes a unitary process that has not yet resolved into a single outcome for him. Both descriptions are generated by ordinary quantum reasoning, but they do not sit comfortably inside one classical frame. If the friend's outcome is already an absolute fact, Wigner's superposition seems incomplete or fictional. If Wigner's superposition is complete, the friend's definite experience seems demoted to something less than a fact. The tension is not psychological; it is formal.

The structural reading begins by refusing to ask too quickly which observer is "right." Instead, it asks what kind of map connects the friend's description to Wigner's description. The friend and Wigner occupy different contexts, and the meaning of a quantum state depends on the context from which it is assigned. The friend's state assignment is conditioned by participation in the measurement interaction. Wigner's state assignment is conditioned by noninteraction with the sealed lab. Their descriptions differ because their physical relations to the measured system differ. The central question is therefore not whether one description must annihilate the other, but how local descriptions are related, transported, compared, or glued when contexts later interact.

2.1. The Friend's Internal Measurement

The friend's role begins with an ordinary measurement. A quantum system is prepared in a superposition relative to some measurement basis. For simplicity, suppose the system may yield either outcome 0 or outcome 1. Before the friend interacts with it, the system is described as a weighted combination of these possibilities. The measurement apparatus is ready, the friend is ready, and the lab is sealed from Wigner's external access. Once the friend measures, the internal laboratory context changes. The apparatus indicates one value. The friend sees that value. A memory is formed. A record may be written. From within the lab, the measurement has produced a definite fact.

This internal definiteness is not superficial. The friend can reason from the result, repeat it aloud, write it in a notebook, and compare it with other internal records. The friend's body, memory, apparatus, and local environment all become correlated with the same outcome. In

ordinary physical terms, the friend's world has been updated. There is no practical ambiguity for the friend about whether a result occurred. A result did occur, and the friend is part of the physical system in which that result is embedded.

The friend's fact may be written as a contextual judgment. It is not simply "outcome 0 exists" in an absolute, context-free sense. It is "within the friend's post-measurement context, there is a record of outcome 0." The record is supported by a physical chain: system, apparatus, perception, memory, and environment. The fact is therefore relationally stabilized. It is not merely a private belief. It is a physical alignment among components of the friend-laboratory system.

This is crucial because the friend's measurement is often trivialized in external accounts. From Wigner's perspective, one may describe the friend as part of a superposition. But from the friend's perspective, the measurement is not waiting to happen. It has happened. The friend's context contains a definite record. Any adequate interpretation must preserve the seriousness of that internal fact while also explaining why Wigner may assign a different state to the unopened lab.

The internal measurement therefore produces a local classicality. It creates a domain in which propositions such as "the pointer reads 0" or "the notebook says 1" become meaningful and definite. Yet this local classicality does not automatically imply global classicality. It does not automatically follow that every outside observer must already have access to, or be forced to assign, the same definite fact before physical interaction connects them to the lab. This distinction between local definiteness and global shareability is the hinge of the entire thought experiment.

2.2. Wigner's External Superposition

Wigner stands outside the sealed laboratory. He knows the initial preparation, the measurement basis, and the rules governing the friend's interaction with the system. However, he has not opened the lab, received a message, observed the apparatus, or interacted with any record of the result. From his external perspective, the lab can be treated as a composite quantum system. The measured particle, the apparatus, the friend's memory, and the written record may all be included in one larger state.

If Wigner applies unitary quantum mechanics to the whole lab, the post-measurement state does not need to be represented as one definite outcome. Instead, it may be represented as a superposition of correlated branches: one in which the system yielded 0 and the friend recorded 0, and another in which the system yielded 1 and the friend recorded 1. Wigner does not describe the friend as having no experience. Rather, he describes the sealed lab as a coherent or potentially coherent combination of alternative friend-record correlations.

This is the disturbing move. The friend says, “I saw a result.” Wigner’s state assignment says, “The lab is in a superposition of result-record correlations.” If one assumes that the quantum state is simply the complete physical reality, then Wigner’s description seems to imply that the friend’s definite outcome is not yet an absolute fact. But if one assumes that the friend’s definite outcome is already absolute, then Wigner’s superposed state seems to be merely epistemic or incomplete. The scenario pressures the meaning of state assignment.

Wigner’s external superposition is not merely ignorance in the ordinary classical sense. In classical probability, Wigner might say, “The friend saw either 0 or 1, but I do not know which.” That statement presupposes a single definite hidden fact. Quantum superposition is structurally different. It permits interference in principle, if the laboratory can be isolated and manipulated coherently. Thus Wigner’s description is not simply a lack of knowledge about a pre-existing classical outcome. It is a different formal description of the entire unopened laboratory.

Of course, whether a macroscopic laboratory could actually be kept coherent is a practical question. Decoherence, environmental leakage, thermodynamic irreversibility, and the complexity of human-scale systems make the literal experiment fantastically difficult. But the conceptual force of Wigner’s friend does not depend on near-term engineering. The point is that if quantum mechanics is universal, then there is no obvious formal boundary at which one must stop applying it. The friend is made of physical systems. The apparatus is made of physical systems. The lab is made of physical systems. Wigner’s external assignment therefore follows from taking the universality of quantum theory seriously.

Wigner’s perspective introduces a second local description. It is local not because it is spatially small, but because it is indexed by Wigner’s interaction history. Wigner has not yet entered the post-measurement correlation structure inside the laboratory. His state assignment reflects that external relation. The problem is therefore not that Wigner denies the friend’s experience. The problem is that Wigner’s physically grounded description does not reduce to the friend’s internal classical record until a new interaction occurs.

2.3. The Conflict Between Definite Records and Unitary Description

The conflict arises because the friend’s internal account and Wigner’s external account appear to support different ontologies. The friend’s account says that measurement has produced a definite record. Wigner’s account says that the entire lab may still be described by unitary evolution. The friend’s fact is outcome-specific. Wigner’s state is branch-inclusive. The friend’s description is classicalized inside the lab. Wigner’s description remains quantum over the lab as a whole.

This conflict is sharpened by the fact that both descriptions are not arbitrary. The friend’s definite record is licensed by ordinary measurement experience. Wigner’s unitary description is

licensed by the quantum treatment of closed systems. The paradox arises because quantum theory seems to license both descriptions in the same physical story. If both are admitted, then the classical assumption of a single absolute fact-space becomes strained.

One way to express the conflict is through levels of description. At the internal level, the measurement basis has become privileged by the apparatus and record. The friend now inhabits one outcome-relative situation. At the external level, Wigner may describe the entire friend-lab system in a basis that preserves the coherence of the global state. The same physical setup therefore has two descriptions that are not simply related by ordinary ignorance. They belong to different contexts and may involve different admissible observables.

A definite record is usually taken to mark the end of quantum ambiguity. Once the apparatus points to 0, the matter seems settled. But Wigner's friend asks: settled for whom, and relative to which interactions? A record inside a sealed lab is settled within the lab. It is not automatically settled for an external observer who has not become correlated with the record. To Wigner, before interaction, the internal record is part of the object being described. To the friend, the record is part of the context from which description occurs.

That reversal is central. What is context for one observer can be object for another. For the friend, the apparatus record is a given fact. For Wigner, the apparatus record is a subsystem within the larger laboratory state. The same element changes role depending on the categorical placement of the observer. This is why the problem is structural rather than merely interpretive. It concerns how descriptions transform when the boundary between observer and observed is redrawn.

The conflict also reveals that "fact" has more structure than classical language admits. A fact may be local, recorded, and operationally usable in one context without yet being globally shared across all contexts. This does not make the fact unreal. It means that objectivity is not primitive; it is constructed through the compatibility of records across interactions. The friend's record becomes public for Wigner only when Wigner interacts with the lab. At that point, a new joint context forms, and the previous descriptions must be reconciled.

The unitary description also has limits. It is not a license to erase the friend's internal world. If Wigner later opens the lab, he does not expect to find a meaningless blur. He expects to find a friend who reports one result and a laboratory record consistent with that report. The external superposition encodes correlations among possible records, not the absence of record-formation. The contradiction emerges only if we demand that one description must be the absolute description before specifying the context of assignment.

2.4. Why the Paradox Is Not Merely Psychological

A common escape is to treat Wigner's friend as a problem about knowledge, consciousness, or subjective experience. On this reading, the friend knows the result while Wigner does not. The paradox then seems to dissolve into ordinary epistemology. But this is insufficient because quantum state assignment is not merely a report of personal ignorance. The external superposition has formal consequences different from a classical probability distribution over unknown outcomes. If Wigner's description is taken seriously, the sealed lab is not merely a box containing one unknown fact; it is a quantum system whose alternative record-states may remain part of a single coherent description.

The problem is also not merely psychological because the friend's record is physical. The friend's memory, the apparatus pointer, the notebook mark, and the laboratory environment are material systems. Their correlations are not private mental impressions. They are part of the physical world. If those correlations are definite inside the lab but superposed from outside, then the issue concerns the physical status of records across contexts, not simply the friend's feeling of certainty.

Nor can the paradox be dismissed by saying that Wigner is simply wrong because the friend already observed the system. That answer assumes the very thing at issue: that observation by the friend creates an absolute fact for all possible observers. If quantum mechanics is universal, then the friend's observation is itself a physical interaction that may be included within a larger quantum description. Declaring the friend's observation final by fiat reinstates a classical cut without explaining why that cut has fundamental authority.

Conversely, the paradox cannot be dismissed by saying that only Wigner's external state is real and the friend's definite experience is somehow illusory. That answer fails to respect the operational reality of internal records. The friend's world contains stable correlations and usable facts. Any account that makes the friend's definite experience unreal in a simple sense risks severing the theory from the very phenomenon it is meant to explain: the occurrence of definite measurement outcomes.

The paradox is therefore structural because it arises from the organization of physical descriptions. There are contexts, records, transformations, and compatibility relations. The friend's context supports one kind of judgment. Wigner's context supports another. The question is how these judgments coexist without contradiction and how they become mutually constrained when interaction occurs. This is exactly the kind of problem that category theory and homotopy type theory are suited to express.

In categorical terms, the friend and Wigner should not be treated as two voices making claims inside a single untyped arena. They should be treated as occupying different objects in a

category of contexts. The measurement, isolation, and later communication are morphisms connecting those objects. In HoTT terms, their facts are not unindexed Boolean assertions but typed judgments whose validity depends on context. Once this structure is made explicit, the apparent paradox becomes a gluing problem: can the local descriptions be assembled into a coherent whole, and under what equivalence conditions?

This reframing does not trivialize Wigner's friend. It makes the problem sharper. The question is no longer whether the friend or Wigner has the "real" state in isolation. The question is what reality must be like if both local descriptions can be physically meaningful and yet no single classical fact table can be assigned prior to the relevant interactions. The thought experiment then becomes a gateway to a broader thesis: quantum foundations should begin not with isolated states, but with the relational architecture that makes state assignments comparable, transportable, and eventually shareable.

3. What Quantum States Can and Cannot Encode

A quantum state is one of the most powerful representational objects in modern physics, but it is also one of the most easily misunderstood. It is tempting to imagine a quantum state as a complete container of information, an infinitely rich physical inscription in which every possible fact about a system is already stored. This temptation is strengthened by the mathematical form of the state itself. A state vector can contain complex amplitudes, relative phases, entanglement structure, and correlations across many degrees of freedom. For a many-body system, the descriptive space grows explosively. It can appear as if the state contains more information than any classical description could ever hold.

Yet the physical meaning of a quantum state is constrained. Not every distinction in the formalism corresponds to a physical distinction. Not every parameter can be measured. Not every feature can be copied. Not every possible value can be made definite at the same time. Not every amplitude can be treated as retrievable classical information. The state is not a transparent database. It is a structure from which possible measurement statistics, correlations, transformations, and records may be derived under specific physical conditions.

This distinction is crucial for Wigner's friend. If one assumes that a quantum state encodes all facts in the same way a classical database encodes entries, then the friend's definite record and Wigner's external superposition seem impossible to reconcile. But if a quantum state encodes context-sensitive possibility structure, relational information, and equivalence-invariant physical content, then the apparent contradiction changes character. The friend's record and Wigner's state assignment need not be treated as rival absolute inventories. They may be treated as

different but related descriptions, each valid within a particular context and each limited by what quantum states can physically encode.

3.1. State Vectors as Physical Descriptions, Not Classical Databases

A classical database is designed to store retrievable entries. If a bit is written into memory, the bit can in principle be read, copied, transmitted, compared, and archived. The stored value is treated as a definite item. A classical database may be incomplete, corrupted, or hidden from a particular user, but its ideal structure is straightforward: entries exist as definite records, and the act of reading them reveals what was already there.

A quantum state does not function this way. A quantum state is not simply a list of classical values waiting to be read. It provides a rule-governed description of preparation, transformation, and measurement. From a state, one can compute probabilities for possible outcomes relative to a chosen measurement context. But the state does not usually assign simultaneous definite values to all possible measurements. It is therefore not a table in which every possible question already has a stored answer.

This difference becomes clear when considering incompatible observables. A system may be prepared so that one observable has a definite value, while another incompatible observable does not. This is not merely a limitation of human ignorance. It is built into the structure of the theory. The state may support precise predictions for one measurement arrangement and only probabilistic predictions for another. The missing value is not simply a hidden classical entry that the observer has failed to inspect. In standard quantum mechanics, it is not part of the state's physical content as a jointly definite fact.

The database analogy also fails because measurement is not passive retrieval. In classical information systems, reading a value can ideally leave the value unchanged. In quantum systems, measurement is an interaction that selects an experimental context and generally changes the state. The result is produced through a relation between system, apparatus, environment, and observer. The quantum state participates in the generation of an outcome, but it is not equivalent to a prewritten outcome-list.

This matters directly for Wigner's friend. The friend's measurement does not merely reveal one entry from a hidden universal table. It creates a physical record within the friend's context. The system, apparatus, and friend become correlated. Inside the lab, this correlation stabilizes a definite outcome. Outside the lab, Wigner may still assign a state to the whole correlated structure. The fact that Wigner's state includes multiple possible friend-record correlations does not mean the friend has no record. It means that Wigner's state is not a classical inventory from the friend's internal standpoint. It is an external quantum description of a larger system.

A state vector is therefore better understood as a compact generator of lawful relations than as a storehouse of completed facts. It encodes how a system may respond to possible interactions. It encodes amplitudes, transition rules, symmetries, and correlations. It may encode entanglement between subsystems. But it does not encode, in classical form, every outcome that every possible observer would regard as already actual. This is the first limit: a quantum state can be complete as a quantum description without being complete as a classical fact-table.

3.2. Global Phase, Gauge Redundancy, and Nonphysical Distinctions

A second limit comes from redundancy. The mathematical expression of a quantum state often contains distinctions that do not correspond to observable physical differences. The simplest example is global phase. Multiplying a state vector by a common phase factor changes the written expression but not the physical state. The vectors may look different on paper, yet they yield the same measurement probabilities and the same physical predictions. The formal distinction is real as notation, but not real as observable content.

This is not a minor technicality. It reveals a general principle: the physical state is not the raw symbol. It is the equivalence class of symbols under transformations that preserve observable structure. If two descriptions differ only by a transformation that changes no possible physical result, then the difference cannot encode physical information. A quantum state cannot store a message in an absolute global phase because there is no operation that reads that phase as a standalone observable. Only relative phases, embedded in possible interference relations, carry physical significance.

Similar issues arise with gauge redundancy, coordinate choices, basis conventions, and representational artifacts. A physical situation may be described in many mathematically distinct ways. The equations may use different bases, different coordinate systems, different gauge choices, or different decompositions of a composite system. These differences can be useful for calculation, but they are not automatically differences in reality. A gauge-dependent feature cannot be treated as a physical fact unless it survives the relevant equivalence relation.

This point is central to an equivalence-first view. Reality is not identical with the most detailed expression of a state in a chosen notation. Reality is what remains invariant under admissible transformations. If a difference disappears under a change of basis, gauge, phase convention, or equivalent representation, then it was not an independent physical fact. The formalism may display it, but the world does not encode it as an observable distinction.

Wigner’s friend is often discussed as if Wigner and the friend simply assign different states. But the deeper question is whether those state assignments are inequivalent physical claims or contextually related descriptions. Some differences between descriptions may be representational. Others may reflect genuine differences in available records or interaction

histories. The task is not to choose the longest or most external description as the real one. The task is to identify the equivalence relations that determine when two descriptions preserve the same physical content.

This also clarifies why a universal quantum state, if such a thing is invoked, cannot be treated as a literal inscription of every formal detail. The state's physical content is not exhausted by its raw coordinates in Hilbert space. Many coordinate-level distinctions are nonphysical. The meaningful object is a structured equivalence class. Thus even before considering measurement, accessibility, or observer-dependence, one must already abandon the idea that every mathematical feature of a state vector encodes an independent fact.

This insight will become important when category theory and HoTT are introduced more fully. In those languages, identity is not mere syntactic sameness. Two presentations may differ and yet be equivalent. Conversely, two superficially similar descriptions may fail to be equivalent if no structure-preserving transformation connects them. Quantum foundations requires this more refined notion of sameness. A state is not merely what is written. It is what can be transported, compared, and preserved under the maps that define physical meaning.

3.3. Measurement, Accessibility, and the Limits of Retrieval

Even when a feature of a quantum state is physically meaningful, it may not be directly retrievable. A quantum state can encode probability amplitudes and correlations, but measurement yields outcomes, not the full state itself. A single measurement on a single system does not reveal the complete amplitude structure. It produces one result relative to one measurement context. To reconstruct a state, one generally requires many identically prepared systems, a chosen reconstruction scheme, and prior assumptions about the system's dimensionality and preparation. This is very different from opening a classical file and reading its contents.

This limit is sometimes obscured by the mathematical richness of quantum states. An arbitrary state of many qubits requires an enormous number of parameters to specify. One might therefore imagine that a quantum system stores an enormous amount of classical information. But the accessible classical information is limited by the rules of measurement. The amplitudes govern outcome statistics, but they cannot all be extracted from a single copy. Unknown quantum states cannot be freely copied, so one cannot simply duplicate the system and measure each copy in a different basis. The no-cloning structure blocks the conversion of unknown quantum information into unlimited classical records.

This is not a defect. It is one of the defining features of quantum information. Quantum states are powerful because they support interference, entanglement, and transformations unavailable to classical systems. But these same features prevent them from behaving as

ordinary readable memory. The state may influence all possible measurement contexts, yet any actual measurement accesses only a limited slice of its structure.

Measurement is also contextual because the choice of measurement arrangement matters. Different observables extract different kinds of information, and incompatible arrangements cannot simply be combined into one universal readout. The state does not hand over all its contents at once. It participates in a specific interaction that produces a specific kind of record. Accessibility is therefore not merely a practical inconvenience. It is part of what the state means operationally.

In Wigner's friend, accessibility separates the friend's internal record from Wigner's external assignment. The friend has access to the outcome because the friend has interacted with the system. Wigner lacks access because he has not interacted with the laboratory record. This is not merely ignorance in the classical sense; it is a difference in physical relation. The friend is inside the correlation structure produced by measurement. Wigner is outside it. The outcome is accessible to the friend as a record, while the whole friend-lab correlation may remain the object of Wigner's quantum description.

Later, when Wigner opens the laboratory, accessibility changes. Wigner becomes correlated with the friend's report and the laboratory record. A new joint context forms. The fact that was internal to the friend's context becomes shareable in a broader context. But this sharing is not magic. It is a physical interaction. Records become public by being copied into new correlations, not by having always existed as entries visible from nowhere.

The limits of retrieval show why quantum states cannot be equated with total readable information. A state may be complete for predicting all possible measurement statistics, yet no observer can extract all those statistics from one instance. A state may describe a laboratory externally, yet the internal friend's record may not be accessible until interaction occurs. Physical information is therefore not just what is formally present in a state. It is what can be accessed, stabilized, and compared through admissible operations.

3.4. The Failure of a Universal Classical Truth Table

The most important limit for the present argument is the failure of a universal classical truth table. A classical truth table imagines that every meaningful proposition about a system already has a definite truth value. The system either has position in this range or not. It either has spin up along this axis or not. It either would produce this outcome or that outcome under each possible measurement. Even if an observer does not know the entries, the entries are presumed to exist.

Quantum theory resists this picture. The state does not generally assign definite values to all possible measurements simultaneously. Contextuality shows that the outcome assigned to a

measurement cannot always be regarded as independent of the larger measurement arrangement in which it is embedded. Incompatible observables cannot be collected into one pre-existing table of values without violating the structure of the theory. The missing table is not hidden behind experimental clumsiness; it is excluded by the way quantum predictions fit together.

Wigner's friend extends this failure from microscopic observables to observer-relative facts. The friend's outcome is definite within the friend's context. Wigner's external state may remain superposed over friend-record correlations. A classical truth-table view demands that there must already be one absolute answer to the question: did the friend see 0 or 1? Quantum theory complicates this demand by allowing different state assignments relative to different interaction histories. The friend's context supports a definite answer. Wigner's unopened-lab context supports a different kind of description.

This does not imply that facts are arbitrary or that contradictions are allowed. It means that truth-value assignment is not context-free. A proposition may require a context in which it is meaningful and decidable. "The friend saw 0" is not just a bare proposition; it is tied to the friend's measurement interaction and record. For Wigner, before interacting with the lab, the corresponding claim may not function as a directly available classical fact. It is embedded in a larger quantum description. When Wigner later interacts with the friend, the proposition becomes part of a shared context and must satisfy compatibility with the friend's record.

Thus the universal classical truth table fails in two related ways. First, quantum systems do not carry simultaneous definite answers to all incompatible measurement questions. Second, observer-involving facts may not be globally shareable before the interactions that connect observers. The world is not a completed spreadsheet viewed from different angles. It is a network of context-dependent records and correlations whose compatibility produces the appearance of a shared classical world.

This failure is precisely where a categorical and HoTT-based account becomes useful. Instead of forcing all facts into one global Boolean table, one can assign facts to contexts and study the maps between them. Some local facts can be glued into global facts. Others cannot. Some descriptions are equivalent under context-preserving transformations. Others reveal genuine obstruction. This is a more faithful structure for quantum reality than the classical image of one total inventory.

The conclusion of this section is therefore not that quantum states are weak or incomplete in a trivial sense. They are not classical databases because reality is not classical at its foundation. Quantum states encode physical possibility, relation, amplitude, and correlation under strict equivalence and accessibility constraints. They cannot encode nonphysical distinctions,

unlimited retrievable information, simultaneous definite values for incompatible observables, or a universal observer-free table of all classical facts. Wigner's friend matters because it forces these limits into the open. If the quantum state is real, its reality is not the reality of a list. It is the reality of a structured, contextual, and equivalence-governed field of possible relations.

4. Observer Contexts as a Category

To move beyond the apparent contradiction in Wigner's friend, we need a language that does not begin by forcing all descriptions into one absolute arena. Category theory offers such a language because it treats meaning as relational from the start. Instead of asking what a thing is in isolation, category theory asks how it is connected, transformed, compared, composed, and preserved across maps. This is exactly the shift required by quantum foundations once observers themselves become physical systems. The friend, Wigner, the laboratory, the measured system, the apparatus, and the written record are not merely objects scattered inside space. They are nodes in a structured network of possible interactions.

A category-theoretic framing begins by treating observational situations as contexts. A context is not merely a mental point of view. It is a physically specified situation: what has interacted with what, which records are available, which measurements have occurred, which systems are isolated, and which transformations are permitted. The friend's context differs from Wigner's because the friend has interacted with the measured system and Wigner has not. This difference is not subjective preference. It is a difference in physical relation.

The category of contexts is therefore a natural stage for Wigner's friend. Each object in the category represents a possible standpoint or physical situation. Each morphism represents an admissible transition, restriction, communication, measurement, embedding, or coarse-graining between contexts. Composition represents the chaining of such physical relations. Identity morphisms represent contexts left unchanged. Once this structure is made explicit, the paradox changes form. The friend and Wigner no longer make incompatible claims in one undifferentiated space. They make different judgments at different objects in a category, and the central question becomes whether these judgments can be transported along morphisms into a common refinement.

4.1. Objects as Physical and Epistemic Contexts

The objects of the relevant category are contexts. A context is a structured domain in which certain descriptions, records, and operations are meaningful. It may include an observer, an apparatus, a preparation procedure, a measurement basis, a memory record, an environmental boundary, or an interaction history. The word "epistemic" is useful, but it must not be

misunderstood. These contexts are not mere private states of belief. They are physical-epistemic situations: what can be known is constrained by what has physically interacted.

In Wigner's friend, the friend's context includes the measured system after interaction, the apparatus state, the friend's memory, and any local record produced inside the lab. The friend can say, within that context, that a definite outcome occurred. The validity of this statement is supported by internal correlations. The apparatus, the friend's perception, and the written note all align around one result.

Wigner's context is different. Before opening the lab, Wigner's context includes the initial preparation, the sealed boundary of the laboratory, and the mathematical description of the friend-lab system as an isolated quantum system. Wigner has not become correlated with the friend's record. The friend's outcome is not available to him as a classical fact. It appears instead as part of the larger state assigned to the unopened laboratory.

The laboratory itself may be treated as another context. It has an internal structure from the friend's perspective and an external boundary from Wigner's perspective. The measured system before interaction is another context. The apparatus before and after measurement may also be represented contextually. Different decompositions are possible depending on the question being asked. Category theory does not require that there be only one privileged object. It allows multiple objects and the maps among them to carry the central content.

This is important because quantum measurement repeatedly changes what counts as background and what counts as object. For the friend, the apparatus record becomes part of the context from which further reasoning proceeds. For Wigner, the apparatus record remains part of the quantum object being described. The same physical subsystem can play different categorical roles depending on where the boundary is drawn. This flexibility is not a weakness. It is exactly what a formal treatment of observer-dependent state assignment requires.

An object in the context category is therefore not a thing in the ordinary material sense. It is a situated domain of description. It specifies what facts are locally available, what operations are meaningful, and what distinctions are physically supported. This gives us a disciplined way to say that the friend's fact is real in the friend's context without immediately forcing it to be a global fact for Wigner before any interaction has connected them.

4.2. Morphisms as Interaction, Measurement, Communication, and Coarse-Graining

If objects are contexts, morphisms are the admissible ways contexts relate. A morphism may represent a physical interaction, a measurement, a communication event, a change of description, a restriction of access, an inclusion of one system into a larger system, or a coarse-graining that forgets microscopic detail. The key is that morphisms preserve enough structure to

make comparison possible. They tell us how a judgment made in one context may be transformed, restricted, or tested in another.

Measurement is one of the most important morphisms. Before the friend measures the system, there is a context in which the system is described as prepared in a superposition. After measurement, there is a new context in which the system, apparatus, and friend's record are correlated. The measurement morphism does not merely reveal a hidden entry. It changes the relational structure of the context. It produces a record and stabilizes a local fact.

Communication is another essential morphism. When Wigner opens the lab and asks the friend what happened, a new relation is formed. The friend's local record is transmitted into Wigner's context. Wigner becomes correlated with the friend's report. The previously external observer now participates in a larger joint context. Communication is therefore not a mere transfer of words. It is a physical operation that changes the category of available facts.

Coarse-graining also matters. A macroscopic record may ignore the detailed quantum microstate of the laboratory while preserving the outcome-relevant structure. The friend does not need complete microscopic knowledge of the apparatus to know that the pointer reads 0. Likewise, Wigner may not need a complete description of every molecular degree of freedom to confirm the friend's report. Coarse-graining is a morphism that forgets some detail while preserving the features relevant to a given level of description.

Restriction maps work in the opposite direction. A larger context may contain more structure than a smaller one. When restricting from a joint Wigner-friend context to the friend's context alone, one forgets Wigner's later record. When restricting from the full laboratory to the measured system, one forgets the apparatus and friend. These restrictions are not arbitrary; they must preserve the relevant relations. If the joint context says that Wigner heard the friend report outcome 0, then restricting to the friend's record should not produce outcome 1 without additional explanation.

The composition of morphisms is central. The system is prepared, then measured by the friend, then later inspected by Wigner, then perhaps reported to a wider scientific community. Each step is a morphism. The whole chain is a composite morphism. Category theory asks whether different paths through this network produce compatible results. If they do, one has a coherent structure of records. If they do not, one has an obstruction.

This is a more precise way to formulate objectivity. Objectivity does not require that every fact be globally available from the beginning. It requires that when contexts are connected by admissible morphisms, their records and descriptions compose coherently. A fact becomes objective not by floating above all contexts, but by surviving transport through the network of relevant physical relations.

4.3. The Friend, Wigner, Laboratory, and System as Related Contexts

The Wigner's friend setup can now be represented as a small category of related contexts. Let one context represent the initial system before measurement. Let another represent the friend inside the laboratory after measurement. Let another represent Wigner outside the sealed lab. Let another represent the joint context after Wigner opens the lab and communicates with the friend. The thought experiment is no longer a story about two isolated observers making incompatible declarations. It is a diagram of contexts connected by physical morphisms.

The initial system context contains the prepared quantum state. At this stage, the relevant description concerns the system and the possible outcomes of a future measurement. The friend has not yet interacted with the system. Wigner may know the preparation procedure. The apparatus is ready but not yet correlated with an outcome. This context supports statements about amplitudes and possible results, not about a completed friend-record.

The friend's post-measurement context contains a definite local record. The measured system, apparatus, and friend are now correlated. If the friend saw outcome 0, then the friend's memory, the apparatus reading, and any local notebook entry align with outcome 0. This context supports outcome-specific judgments. The friend may reason as someone who has seen a result.

Wigner's external context contains the sealed laboratory as an object of quantum description. Wigner does not have access to the internal outcome record. If he treats the lab as isolated, he may assign a superposed state over correlated friend-record alternatives. This context supports judgments about the entire unopened lab. It does not support direct assertion of one internal outcome as Wigner's own record.

The joint context after communication contains both Wigner and the friend in relation. Wigner has opened the lab, the friend has reported the outcome, and Wigner has become correlated with the laboratory record. This new context is not identical to either prior context. It refines them by adding a shared relation. The friend's local fact can now be transported into Wigner's record. Wigner's previous external state assignment must now be updated or reexpressed relative to the new interaction.

The category-theoretic diagram makes clear that the issue is not whether the friend's context or Wigner's context should be deleted. Both are legitimate nodes in the structure. The issue is how they connect. Before communication, there may be no morphism that gives Wigner direct access to the friend's definite record. After communication, there is such a morphism. The availability of facts depends on the existence and character of these maps.

This also clarifies why the laboratory boundary matters. The sealed lab is not merely a narrative device. It defines the absence of a certain class of morphisms. No signal, record, or

environmental leakage is supposed to connect the friend's internal outcome to Wigner. The sealed boundary preserves Wigner's ability to treat the lab as a quantum object. Opening the lab creates new morphisms. The boundary is therefore categorical: it determines which maps are present or absent in the diagram.

In this diagrammatic view, the apparent paradox comes from collapsing several contexts into one. If we erase the distinction between the friend's post-measurement context and Wigner's pre-communication context, we are forced to compare judgments that were never licensed in the same domain. The friend's "I saw 0" and Wigner's "the lab is in a superposition of record-correlations" become contradictory only after their context labels are stripped away. Restoring the diagram does not automatically solve every interpretive problem, but it prevents a false contradiction produced by untyped language.

4.4. Context Change as a Physical Operation

A crucial feature of this approach is that context change is not merely a change in description. It is a physical operation. When the friend measures the system, the context changes because a physical interaction has occurred. When Wigner opens the lab, the context changes because new correlations are formed. When a record is copied, erased, decohered, protected, or communicated, the context changes because the physical possibilities of later comparison have changed.

This prevents the categorical approach from becoming a loose metaphor for perspective. Contexts are not arbitrary frames that observers can adopt at will. They are constrained by real interactions. The friend cannot simply choose to occupy Wigner's external context after having measured the system. Wigner cannot simply choose to possess the friend's record before interacting with the lab. Contexts are determined by the physical maps that have and have not occurred.

The measurement context is changed by entanglement and record formation. The communication context is changed by information transfer. The public scientific context is changed when multiple observers compare records. Each change modifies the available judgments. A proposition that was meaningful and decidable in one context may be unavailable, undecidable, or differently represented in another. This is not relativism. It is the recognition that facts have physical preconditions.

Context change also has a temporal dimension, though it is not reducible to ordinary time ordering. A measurement may occur before Wigner opens the lab, but the relevant issue is not simply earlier versus later. The issue is which systems have become correlated. The friend may have a record at one stage; Wigner may not. After interaction, Wigner does. The growth of shared facts follows the growth of physical correlations.

From an equivalence-first perspective, context change asks what is preserved. When moving from the friend's internal context to the later joint context, the friend's record should be preserved if no disturbing operation intervenes. When moving from Wigner's external context to the later joint context, Wigner's superposed assignment is not preserved as a direct classical statement; it is transformed by interaction. The physical content of a description is whatever can be transported coherently along the relevant morphisms.

This point is especially important for interpreting collapse. In the categorical view, collapse need not be treated immediately as a mysterious global event. It can be treated as a change in context: a transition from a description in which multiple outcomes are represented in amplitude structure to a description in which a particular record is available. Whether one gives this transition an ontological, epistemic, relational, or branching interpretation is a further question. But the minimal structural fact is that the context of valid judgments has changed.

Context change also explains why objectivity is cumulative. A private record becomes interpersonal when communicated. An interpersonal record becomes scientific when stabilized across instruments, observers, and repetitions. A laboratory fact becomes public not because it was accessible from nowhere all along, but because it survives a network of context changes. The stronger the network of compatible morphisms, the more objective the fact becomes.

The Wigner's friend scenario is therefore best understood not as a contest between two mental viewpoints but as a diagram of physical context changes. The friend's measurement, Wigner's isolation, the laboratory boundary, and later communication are all structural elements. Category theory provides the grammar for this structure. It allows us to say that quantum reality is not located in one context alone. It is distributed across a coherent pattern of contexts and the morphisms that connect them.

5. Quantum State Assignment as a Presheaf

Once observer contexts are treated categorically, the next step is to ask what kind of mathematical object a quantum state assignment should be. The classical instinct is to place one state at the top of the hierarchy and treat all observer descriptions as partial views of that single object. But Wigner's friend suggests a different structure. Instead of beginning with one absolute state that all observers must share, we may assign state descriptions to contexts. Each context receives the state description appropriate to its interaction history, available records, and admissible operations. The resulting structure is not a single state but a context-indexed family of states.

This is where the presheaf idea becomes useful. A presheaf assigns data to contexts and assigns maps between those data whenever contexts are related. In the present setting, each

observational context receives a space of possible state descriptions, records, propositions, or judgments. A morphism between contexts induces a corresponding translation, restriction, or update relation between the associated state descriptions. Thus the friend's internal state assignment, Wigner's external state assignment, and the later joint state assignment are not forced into one flat arena. They become sections of a structured assignment over a category of contexts.

The presheaf view captures a basic feature of quantum practice. State assignment is not made from nowhere. It is made relative to preparation, measurement basis, isolation assumptions, environmental access, and available records. When those conditions change, the state assignment changes. The change is not arbitrary. It must follow rules. Those rules are the maps of the presheaf. In this sense, the presheaf is not merely a technical decoration. It formalizes the idea that quantum states are contextual objects whose meaning is governed by how they transform across physical situations.

In Wigner's friend, this means that the friend's post-measurement context is assigned a state or record structure in which a definite outcome is available. Wigner's pre-communication context is assigned a state in which the entire unopened lab may remain superposed. The later Wigner-friend context is assigned a state or record structure in which the friend's report and Wigner's observation become mutually correlated. The theory does not require that these assignments be identical. It requires that they relate coherently when the relevant context maps exist.

5.1. Assigning State Spaces to Contexts

A presheaf begins by assigning a type of data to every context. In quantum foundations, that data may be a Hilbert space, a density operator, a set of possible records, a probability assignment, a family of observables, or a HoTT-style type of judgments. The exact choice depends on the level of formalization. What matters for the present paper is the structural principle: each context has its own admissible state space.

The friend's context has a state space shaped by the measurement already performed. It includes outcome-indexed records. If the friend saw 0, the relevant local description includes a friend-memory state corresponding to 0, an apparatus state corresponding to 0, and perhaps a written record corresponding to 0. If the friend saw 1, the corresponding local description contains the 1-record structure. From the friend's perspective, the post-measurement state space is not an abstract superposition of unexperienced possibilities. It is organized around the actual record available in that context.

Wigner's context has a different state space. Before communication, Wigner treats the lab as a system enclosed behind a boundary. His admissible descriptions include the full friend-lab quantum state, possibly written as a superposition of correlated record-branches. The friend's

record is not directly available to Wigner as a local datum. It is represented within the larger state he assigns to the sealed laboratory. Wigner's state space therefore includes descriptions that the friend would not use for the friend's own internal experience.

The joint context after communication has still another state space. It includes Wigner's record of the friend's report, the friend's continuing record, and the laboratory evidence tying them together. The state assignment appropriate to this context is not simply Wigner's earlier external superposition, nor is it merely the friend's isolated record. It is a new context in which both observers are mutually correlated around a shared fact.

This context-indexed assignment avoids a common confusion. Different contexts need not be treated as competitors for one privileged state. They are domains in which different state spaces are appropriate. A microscopic preparation context, a friend's measurement context, Wigner's external context, and a later shared communication context are not interchangeable. They support different possible judgments because they contain different physical relations.

The assignment of state spaces to contexts also allows scale to be handled flexibly. A context may be microscopic, macroscopic, local, extended, internal, external, or communal. Nothing in the categorical framework requires a sharp metaphysical boundary between quantum and classical systems. A classical-looking record may appear as a stable structure within one context and as part of a larger quantum system within another. This is exactly the flexibility required by Wigner's friend.

The central point is that a quantum state assignment is not nakedly attached to a system alone. It is attached to a system-in-context. The same physical subsystem may receive different descriptions depending on whether it is treated as measured, unmeasured, isolated, decohered, communicated, or embedded in a larger experimental arrangement. A presheaf captures this dependence by placing state descriptions over contexts rather than pretending that every observer must use the same description at every stage.

5.2. Restriction, Translation, and Update Maps

A presheaf does not merely assign data to isolated contexts. It also assigns maps between the data associated with related contexts. These maps are essential because they determine how descriptions move when contexts are connected. In the Wigner's friend scenario, relevant maps include restriction, translation, update, communication, embedding, and coarse-graining. Each map expresses a different way in which one context may be related to another.

Restriction occurs when a richer context is reduced to a poorer or narrower one. Suppose there is a later joint context containing both Wigner's record and the friend's record. Restricting this context to the friend alone forgets Wigner's later observation. Restricting it to Wigner alone forgets the friend's private internal sequence except insofar as it has been communicated.

Restricting it to the apparatus alone forgets the human observers and retains only the pointer state. These restrictions must preserve the relevant compatible record structure. If the joint context contains a stable shared record of outcome 0, restriction to the apparatus should not yield an outcome-1 pointer without a further physical process explaining the discrepancy.

Translation occurs when descriptions are reformulated across representational schemes. A state may be written in one basis or another. A density matrix may replace a pure-state description for a subsystem. A record may be described microscopically, macroscopically, informationally, or experientially. Translation maps are not arbitrary changes of language. They must preserve the physical content relevant to the context. If two descriptions are related by a valid translation, their difference is representational rather than ontological.

Update occurs when a new interaction changes the context. When Wigner opens the lab, his earlier state assignment is not simply carried forward unchanged. It is updated in light of the new correlation with the friend and laboratory record. The update map connects Wigner's pre-communication context to Wigner's post-communication context. It is a physical and informational transition, not a purely mental act. Wigner becomes part of the correlation structure he previously described from outside.

Communication maps are especially important. The friend's record becomes available to Wigner only through a physical process: speech, writing, signal transmission, instrument readout, or some other interaction. The communication map carries a local fact into a broader context. It does not create objectivity by decree. It creates a new correlation that allows a previously local record to become shared.

Embedding maps move in the opposite direction. The friend's laboratory can be embedded in Wigner's larger description. The friend's record, which is contextual background for the friend, becomes a subsystem inside Wigner's object-level state assignment. This embedding explains why the same record can be both definite internally and part of an external quantum description. The role of a record depends on where it sits in the diagram.

Coarse-graining maps forget microscopic detail while preserving macroscopic structure. The exact quantum microstate of the notebook may be irrelevant to the fact that it contains a mark corresponding to outcome 0. The exact molecular state of the friend's brain may be irrelevant to the fact that the friend remembers 0. Coarse-graining allows classical records to emerge as stable equivalence classes of many microscopic realizations. This is crucial for connecting quantum state descriptions with ordinary facts.

Together, these maps discipline contextuality. Without maps, context-dependence would collapse into unrelated private worlds. With maps, context-dependence becomes structured. State assignments can differ, but they must transform coherently when contexts are related. The

presheaf is therefore a formal compromise between absolutism and subjectivism. It rejects one universal classical truth table, but it also rejects arbitrary observer-private reality. What matters is how state assignments restrict, translate, update, and glue.

5.3. Why Different Observers May Assign Different States

The presheaf view explains why different observers may assign different quantum states without producing immediate contradiction. State assignment depends on context, and observers may occupy different physical contexts. The friend has interacted with the measured system. Wigner has not. The friend has access to a definite record. Wigner does not. The friend's state assignment and Wigner's state assignment differ because the physical relations grounding their descriptions differ.

This is not the same as saying that each observer invents a private world. The friend's record is physically grounded. Wigner's external superposition is physically grounded. Each assignment reflects a real relation to the system. The difference lies in the absence, before communication, of a context in which both descriptions must already be expressed as one shared classical fact. Their later interaction creates such a context.

In classical physics, different observers may assign different probability distributions because they have different information. But classical probability usually presupposes an underlying definite state. One observer may know the coin landed heads while another does not, but the coin has already landed one way. The ignorant observer's probability is merely epistemic. Quantum state difference is more subtle. Wigner's assignment to the sealed lab is not necessarily just ignorance about a pre-existing classical fact. It may be a legitimate quantum description of the friend-lab system as an unobserved whole.

This is why Wigner's friend is stronger than a story about hidden information. If the laboratory remains isolated in principle, Wigner's external state may include coherent alternatives. Such a description is structurally different from classical uncertainty. The friend's local record and Wigner's global assignment are not simply two knowledge states about the same underlying classical event. They are different context-indexed descriptions.

The presheaf view preserves the friend's authority within the friend's context. The friend does not need Wigner's permission for the measurement to have occurred internally. The apparatus points, the memory forms, the record exists. At the same time, the presheaf view preserves Wigner's authority within Wigner's context. Before interacting with the lab, Wigner is not required to assign the same definite record as a local fact. His description may treat the lab as a quantum system.

Contradiction would arise only if the two descriptions were asserted in the same context with incompatible content. For example, if the joint Wigner-friend context contained a stable friend-

record of 0 and a stable Wigner-record that the friend reported 1 under conditions that should preserve communication, then a genuine inconsistency would need explanation. But prior to the formation of the joint context, the friend's and Wigner's descriptions are not judgments of the same type in the same domain.

This helps clarify the meaning of observer-relative facts. A fact relative to an observer is not a whim, opinion, or psychological certainty. It is a physically indexed judgment. The friend's fact is relative because it belongs to a context defined by interaction with the system. Wigner's superposition is relative because it belongs to a context defined by noninteraction with the sealed lab. Relativity here means dependence on physical relation, not dependence on arbitrary belief.

Different state assignments can therefore coexist when they occupy different fibers over the context category. The presheaf does not require these assignments to be identical. It requires them to be compatible under the maps that actually exist. Before Wigner opens the lab, certain maps are absent. After Wigner opens the lab, new maps exist, and compatibility conditions become stronger. This is the structural heart of the Wigner's friend problem.

5.4. Presheaves Versus Absolute State Realism

Absolute state realism treats the quantum state as a single observer-independent object corresponding directly to physical reality. On this view, there is one true state of the system, and different observer assignments are either approximations, ignorances, or partial descriptions of that one state. This approach is attractive because it promises a clear ontology. It seems to say what exists: the quantum state exists.

But Wigner's friend pressures absolute state realism. If Wigner's external state is the one true state, then the friend's definite internal record seems difficult to interpret. Is the friend's experience merely branch-relative, emergent, or not yet absolute? If the friend's post-measurement state is the one true state, then Wigner's unitary external description seems demoted. Is it only ignorance, despite its quantum form? Absolute state realism must choose how to privilege one description or reinterpret the other.

The presheaf view avoids the need for immediate privilege. It does not deny that state assignments can be physically meaningful. It denies that their meaning is exhausted by one absolute, context-free object. A state assignment belongs to a context. The full structure is not one state but the organized family of assignments across contexts and the maps connecting them. Physical reality lies not in a single isolated fiber but in the coherence of the whole presheaf-like structure.

This does not mean the presheaf view must reject all forms of realism. It rejects a simple realism of raw state vectors. It supports a realism of structure, relation, and equivalence. The

real object is the pattern of possible assignments and transformations, the compatibility conditions among records, and the invariants that survive across context changes. This is a more sophisticated realism, not an abandonment of reality.

Absolute state realism often struggles with representational redundancy. If the raw state vector is real, what about global phase? What about gauge freedom? What about basis dependence? What about equivalent mathematical presentations? The presheaf view naturally treats such features as part of the representation unless they survive the relevant equivalence maps. The physically real content is what is preserved structurally, not every coordinate-level feature of a chosen description.

The presheaf view also gives a natural account of objectivity. Objectivity is not the possession of one view from nowhere. It is the stable agreement of descriptions across admissible maps. A fact becomes objective when it can be transported, restricted, communicated, and compared without contradiction across a sufficiently rich network of contexts. This matches scientific practice more closely than the fantasy of an observer-free database. Scientific facts are stabilized through instruments, records, replications, communications, and shared transformations.

In Wigner's friend, the presheaf view says that before Wigner opens the lab, the friend's context and Wigner's context support different assignments. After Wigner opens the lab, a joint context forms in which their records must cohere. The transition from local fact to shared fact is a gluing process. If the local assignments can be glued, a broader fact emerges. If they cannot, the obstruction signals nonclassical structure.

Thus the presheaf view reframes the ontology of quantum states. A quantum state is not a universal container of all facts. It is a context-indexed assignment within a larger relational structure. The central object of quantum foundations is not the state alone but the presheaf of state assignments over contexts, together with the maps, equivalences, and gluing conditions that make shared reality possible.

6. Measurement as Morphism Rather Than Primitive Collapse

The measurement problem is often presented as a crisis caused by two incompatible rules. One rule says that quantum systems evolve smoothly and deterministically according to unitary dynamics. The other says that measurement produces a definite outcome, apparently selecting one result from among many possibilities. The first rule is continuous, reversible, and branch-preserving. The second appears discontinuous, irreversible, and outcome-selecting. Wigner's friend intensifies the problem by allowing both rules to be applied to the same event from

different contexts. For the friend, measurement has produced a definite record. For Wigner, the friend's measurement can be included in a larger unitary description of the sealed laboratory.

A categorical reframing changes the starting point. Instead of treating collapse as a primitive metaphysical event, measurement may be treated first as a morphism between contexts. A measurement is a physical map that changes the relational structure among system, apparatus, record, observer, and environment. It does not merely reveal a pre-existing classical value, nor does it need to be described immediately as an unexplained global jump. It creates correlations, establishes records, privileges certain distinctions, and makes new judgments available within a context. Measurement is therefore an operation that transforms one domain of description into another.

This does not eliminate the interpretive question of collapse. It relocates it. The first question is not "At what exact instant does the wavefunction collapse?" but "What contextual map has occurred, and what structure does it preserve or create?" The friend's measurement is a morphism from a preparation context to a record-bearing context. Wigner's external unitary account is a morphism from an initial sealed-lab context to a later sealed-lab context in which the system, apparatus, and friend are correlated. These are different descriptions of related structural transformations. The apparent contradiction arises when they are forced into a single context without tracking the maps that generate them.

6.1. The Measurement Interaction as a Structure-Preserving Map

A measurement begins with a system prepared in a state that may yield more than one possible outcome relative to a chosen basis. The apparatus is arranged to distinguish those outcomes. Before the interaction, the system and apparatus are not yet correlated in the relevant outcome-bearing way. After the interaction, the apparatus state depends on the system state. If the system is found as 0, the apparatus points to 0. If the system is found as 1, the apparatus points to 1. At the level of relational structure, the measurement creates a map from possible system states to possible record states.

This map is structure-preserving because it is not an arbitrary disturbance. A competent measurement preserves the relevant distinction it is designed to detect. If the apparatus is measuring a two-outcome observable, then its pointer states must reliably correspond to those outcomes. The measurement interaction must preserve enough information about the chosen observable to support later comparison, memory, and communication. A failed measurement is precisely one in which this structure is not preserved.

In a simplified quantum description, the measurement interaction correlates system alternatives with apparatus alternatives. A system component associated with outcome 0 becomes correlated with an apparatus record of 0. A system component associated with

outcome 1 becomes correlated with an apparatus record of 1. The key operation is not the magical appearance of a number from nowhere. It is the creation of a stable relation between the measured system and a record-bearing device.

For the friend, this interaction continues beyond the apparatus. The friend looks at the pointer. The friend's perceptual state becomes correlated with the apparatus state. The friend's memory becomes correlated with the perceptual state. If the friend writes the result down, the notebook becomes correlated as well. The measurement morphism therefore extends through a chain of record-producing interactions. The system does not merely interact with a machine; it enters a broader web of physical correlations.

This web is what makes an outcome usable. A pointer flash that immediately disappears without leaving any trace would not function as a stable measurement record. A record must be capable of supporting future judgments. It must be preserved long enough, and in a robust enough form, to be accessed by some later process. Measurement as morphism therefore includes not only correlation but also stabilization. The map must create a structure that can survive relevant context changes.

The phrase "structure-preserving" must also be understood relative to the chosen measurement context. A measurement designed to reveal one observable may disturb another. It preserves the distinctions relevant to the selected basis while failing to preserve incompatible distinctions. This is one of the reasons quantum measurement cannot be treated as passive reading. The measurement morphism is selective. It creates a record of one kind of distinction while transforming the system in a way that may erase or obscure others.

In Wigner's friend, the internal measurement morphism produces a record-bearing context for the friend. This is not in dispute. The friend's world after measurement is structurally different from the friend's world before measurement. The apparatus, memory, and local environment now encode an outcome. The categorical question is how this internal morphism relates to Wigner's external morphism, in which the entire sealed laboratory is treated as evolving without a primitive collapse.

6.2. Record Formation Inside the Friend's Context

Inside the laboratory, measurement becomes real as record formation. The friend does not encounter an abstract state vector. The friend encounters an apparatus reading, a perceptual event, a memory, and perhaps a written mark. These are not merely subjective impressions. They are physical inscriptions distributed across the friend-laboratory system. The fact that the friend saw 0 or 1 is embodied in correlated material states.

Record formation is the local birth of classicality. Before measurement, the system may be described as a superposition relative to the measurement basis. After measurement, the

friend's context contains a record that selects one outcome. The friend does not reason from amplitudes alone. The friend reasons from the actual pointer reading. This is why the friend can say, without hesitation, that a result occurred. The friend's context has changed from a context of possible outcomes to a context of recorded outcome.

The record is not isolated in one place. A robust record is redundant. The apparatus points, the friend remembers, the notebook may contain a mark, and the immediate environment may carry traces of the interaction. Each additional correlation makes the record harder to erase and easier to communicate. This redundancy is what allows a local measurement fact to become stable. A single microscopic correlation may be fragile, but a macroscopic record is supported by many mutually reinforcing physical states.

From the friend's perspective, collapse is an effective description of this transition. Before looking, the friend may assign probabilities to possible outcomes. After looking, the friend conditions on the observed result. The friend's future predictions, expectations, and actions are updated accordingly. In the friend's context, the alternatives are no longer equally live as personal records. One outcome has been registered.

The categorical view does not deny this effective collapse. It explains it as a transition between contexts. The friend moves from a pre-measurement context to a post-measurement context. The post-measurement context contains a witness of one outcome. In HoTT-like language, a type corresponding to "the friend saw 0" or "the friend saw 1" has become inhabited within that context. The fact is not a detached Boolean value; it is supported by a concrete witness in the friend's physical situation.

This also explains why the friend's fact should not be dismissed as merely apparent. A recorded outcome is a physical achievement. It is what experimental science depends on. If all records were treated as unreal until certified by an external observer, no measurement would ever begin. The friend's record is real as a local fact because it is available for local reasoning, memory, and action.

At the same time, local record formation does not automatically settle every external description. Wigner has not yet become correlated with the friend's record. The record exists inside the laboratory context, but Wigner has not accessed it. If the lab is perfectly sealed, Wigner's context lacks the morphism needed to import the friend's record as a Wigner-available classical fact. Thus the same record can be internally definite while externally represented as part of a larger quantum object.

This is the key distinction. A record can be definite in the context in which it has formed without being globally available to every context. The friend's measurement morphism creates an outcome-bearing context. It does not by itself create a universal context in which all possible

observers have already acquired the outcome. Public objectivity requires further morphisms: communication, comparison, and shared stabilization.

6.3. Unitary Evolution in Wigner's Context

From Wigner's external standpoint, the sealed laboratory can be treated as one composite physical system. If Wigner assumes that the lab is isolated from him and from the outside environment, then he may apply unitary evolution to the whole system: measured particle, apparatus, friend, memory, notebook, and internal environment. In this description, the friend's measurement is not a primitive collapse. It is an interaction that entangles the system with successive record-bearing degrees of freedom.

This description does not say that nothing happened inside the laboratory. Something very important happened: correlations formed. But Wigner represents those correlations without selecting one outcome as his own record. The external state may include a component in which the friend saw 0 and a component in which the friend saw 1. These components are not independent classical possibilities in the ordinary sense. They are terms in a quantum description of the unopened lab.

The Wigner-context description is valid because Wigner has not interacted with the internal record. The sealed boundary prevents the friend's outcome from entering Wigner's available classical facts. If no information leaks out, then Wigner's state assignment is governed by the dynamics of the whole isolated lab rather than by a received measurement result. His description is therefore not simply stubbornness or ignorance. It follows from his physical relation to the laboratory.

This unitary description treats the friend's record as object rather than context. For the friend, the memory of the outcome is part of the background from which reasoning proceeds. For Wigner, that memory is part of the system being described. The same physical structure appears in different roles. This role reversal is one of the main reasons Wigner's friend is so powerful. It shows that "record" is not an absolute category independent of context. A record for one system may be a quantum subsystem for another.

Unitary evolution in Wigner's context also preserves coherence in principle. If the laboratory could be manipulated with impossible precision, Wigner might imagine reversing the measurement interaction or testing interference between different record-branches. In practice, macroscopic decoherence makes such control fantastically difficult. But the conceptual possibility is enough to distinguish Wigner's quantum description from ordinary classical uncertainty. The sealed lab is not merely a box with a hidden slip of paper. It is, in the formal description, a composite quantum system.

This creates the familiar tension. The friend's context contains an effective collapse into one record. Wigner's context contains unitary correlation across possible records. Both are generated by legitimate physical reasoning from their respective standpoints. The categorical approach does not immediately force one to cancel the other. It asks how the two descriptions are related by context maps.

When Wigner opens the lab, his context changes. The unitary external description is no longer the appropriate state assignment for Wigner as an uncorrelated outside observer, because Wigner is no longer outside the record structure. He becomes part of it. A communication morphism connects him to the friend's internal record. His state assignment must then be updated to the new joint context. The earlier unitary description was valid for the sealed-lab context; it is not a timeless statement that must remain unchanged after interaction.

This is why measurement should be understood through context rather than as a single mysterious event from nowhere. In Wigner's context, the friend's measurement is a unitary correlation-producing process. In the friend's context, it is a record-producing outcome event. These are not necessarily contradictory because they are not the same judgment in the same context. The contradiction appears only if one assumes that there must be a single context-free description that all observers must already share.

6.4. Collapse as a Projection of Context, Not a Universal Event

The categorical reframing suggests that collapse may be understood, at least minimally, as a projection of context rather than an unexplained universal event. This does not mean that all interpretations must become purely epistemic. It means that the first structural role of collapse is to describe a transition from a context containing multiple possible outcomes to a context containing a definite accessible record. Collapse marks the updating of admissible judgments when a record becomes available.

For the friend, collapse occurs because the friend now has a record. The pre-measurement alternatives are no longer all compatible with the friend's actual memory and apparatus reading. The friend's context has projected onto an outcome-bearing branch of description. This projection is not merely a calculation. It reflects a physical change in the friend-lab system: record formation. The friend's future predictions are conditioned on the recorded result.

For Wigner before opening the lab, that projection has not occurred in Wigner's context. Wigner has no outcome record. He may still assign a superposed state to the unopened lab. If collapse were an absolute global event, then the friend's measurement would force Wigner's state assignment to collapse immediately, regardless of any physical relation between Wigner and the lab. But the Wigner's friend scenario exists precisely because quantum theory does not make that global event easy to locate without additional interpretive commitments.

Treating collapse as context projection avoids premature metaphysics. It says that collapse is the change in state assignment appropriate to a context after a record becomes available in that context. This is compatible with several deeper interpretations. An objective-collapse theorist may add that a real physical collapse occurs under certain conditions. A many-worlds theorist may say that no global collapse occurs, only branch-relative updating. A relational theorist may say that facts are relative to interacting systems. A QBist may say that collapse is an agent's updating of expectations. The categorical account identifies the shared structural feature: collapse is tied to context change and record availability.

This helps dissolve a false dilemma. We need not say either that the friend's outcome is globally absolute at once or that it is unreal. We can say that the friend's outcome is a real record in the friend's context, while Wigner's pre-communication context does not yet contain that record as an accessible fact. When Wigner interacts with the lab, the relevant projection occurs for Wigner as well. The local record becomes part of a larger shared context.

Collapse as context projection also explains why the same physical event can be described differently without contradiction. The measurement interaction produces correlations. In one context, those correlations support a definite outcome. In another context, they remain part of a larger unitary state assignment. The difference lies not in arbitrary choice but in the physical placement of the observer relative to the record.

The projection metaphor is especially useful because it emphasizes loss and selection. A post-measurement context does not retain all the symmetries and possibilities of the pre-measurement context as live local alternatives. Once the friend has seen 0, the friend cannot honestly treat "I saw 1" as equally compatible with the present record. The context has narrowed. But this narrowing is local to the record-bearing context. It need not be imposed on every external description unless a morphism carries the record outward.

Thus measurement is not a primitive collapse imposed from above. It is a morphism that creates records, followed by context-dependent projections that determine which judgments become available. This approach retains the seriousness of definite outcomes while avoiding the need to declare a single observer-independent collapse boundary at the outset. It converts the measurement problem into a problem of maps, records, and context transitions.

The lesson for Wigner's friend is direct. The friend's measurement is a morphism into a record-bearing context. Wigner's external unitary description is a morphism in a sealed-lab context. Collapse appears when a context acquires an outcome record and updates its admissible judgments. The deeper reality is not a single dramatic jump in a universal state vector, but the structured relation among contexts as records are formed, preserved, communicated, and glued into shared objectivity.

7. Pullbacks, Communication, and Shared Facts

If measurement creates local records, communication creates shared facts. Wigner's friend becomes most revealing at the moment when Wigner opens the laboratory and the friend reports what was seen. Before that moment, the friend's context and Wigner's context support different state assignments. The friend has an internal outcome record. Wigner has an external description of the unopened laboratory. These descriptions are not identical, but they need not yet be contradictory because they belong to different domains of access. The decisive question is what happens when a new physical relation joins the two domains.

Category theory gives a precise language for this joining. When two contexts are brought into relation through a common refinement, the appropriate structure is not simple addition but a pullback-like construction. A pullback forms a context in which two previously separated descriptions are compared over something shared. In Wigner's friend, the shared object may be the laboratory event, the apparatus record, the friend's report, or the physical history that both observers now reference. The joint Wigner-friend context is the place where the friend's internal fact and Wigner's external description must become mutually compatible.

This is the birth of public fact. A public fact is not simply a private record made louder. It is a record that has survived transport into a larger context. The friend's statement, the apparatus reading, Wigner's observation, and any later transcript must align under the relevant maps. If they do, the outcome becomes stabilized as a shared fact. If they do not, the discrepancy must be explained by error, disturbance, deception, memory failure, apparatus failure, decoherence, reversal, or some other physical process. The shared fact is therefore not a primitive atom. It is a relational invariant across a network of compatible records.

7.1. Opening the Laboratory as a Contextual Refinement

Opening the laboratory is often described casually, as if Wigner simply looks inside and learns what was already there. But in the categorical account, opening the lab is a physical operation that refines the context. Before opening, Wigner's context excludes direct access to the friend's outcome record. The sealed boundary prevents the friend's internal fact from becoming Wigner's fact. After opening, the boundary is crossed. Wigner becomes physically correlated with the friend, the apparatus, the notebook, and the laboratory environment. A new context is formed.

This new context is not merely Wigner's old context plus one item of information. It is structurally different. Wigner is no longer external to the laboratory record. He has become part of the record-bearing network. The friend's memory and report, which were previously internal to the laboratory, now stand in relation to Wigner's perception and memory. The outcome can

now be described as shared between them. The operation of opening the lab therefore changes what judgments are admissible.

The term refinement is appropriate because the new context contains more relational structure than either context alone. The friend alone has the internal outcome. Wigner alone has the external sealed-lab description. The joint context contains the relation between Wigner and the friend's reported outcome. It allows questions that were previously unavailable, such as whether Wigner's observation agrees with the friend's memory, whether the notebook agrees with the apparatus, and whether the later report agrees with the original record.

A contextual refinement does not erase the earlier contexts. It organizes them. The friend's earlier context remains the context in which the record first formed. Wigner's earlier context remains the context in which the sealed lab could be assigned an external state. The joint context relates these earlier descriptions through new physical maps. It does not make them identical in retrospect; it makes them comparable.

This is important because much confusion arises from treating later shared facts as if they must have had the same status in every prior context. Once Wigner hears the friend say "I saw 0," it is tempting to conclude that "the friend saw 0" must have been a Wigner-available classical fact all along. But the categorical picture distinguishes existence of a local record from availability in a broader context. The friend's record was locally definite before Wigner opened the lab. It became jointly definite for Wigner only after the communication morphism connected them.

Opening the laboratory also changes Wigner's state assignment. The earlier sealed-lab superposition was appropriate to Wigner's noninteraction with the lab. Once Wigner interacts, he no longer occupies that context. His description must be updated to the joint context. This update is not an arbitrary mental correction. It is required by the new physical correlation. The external observer has become an internal participant in a larger record structure.

Thus opening the lab is a contextual refinement because it creates a new object in the category of contexts: the post-communication Wigner-friend context. This object maps back to the friend's record context and to Wigner's external context, but it contains relations absent from both. The paradox softens when we recognize that shared objectivity appears at this refined object, not automatically at every prior node of the diagram.

7.2. Pullbacks as Common Worlds of Agreement

A pullback is a universal construction that captures the idea of two structures agreeing over a common base. In ordinary terms, it forms the space of pairs that are compatible with respect to some shared reference. In the Wigner's friend setting, the pullback-like joint context contains compatible pairs of descriptions: what the friend recorded and what Wigner later observes,

what the apparatus indicated and what the friend reports, what the laboratory history supports and what later observers can confirm.

The value of the pullback metaphor is that it avoids two extremes. On one extreme, one might say that the friend's fact and Wigner's description are simply unrelated private worlds. On the other extreme, one might say that they must already be the same fact in the same way before any interaction connects them. The pullback gives a third option. The descriptions become jointly constrained when they are mapped into a common context of comparison.

In this common context, agreement is not merely verbal. It is physical compatibility. If the friend says outcome 0, the notebook says outcome 0, the apparatus trace indicates outcome 0, and Wigner records hearing outcome 0, then these records form a compatible family. The shared fact is the invariant supported by that family. It is not located solely in the friend's mind, solely in Wigner's mind, or solely in the apparatus. It is distributed across the agreement structure.

The pullback also explains why disagreement is meaningful. If Wigner opens the lab and the friend reports 0 while the notebook says 1, the joint context does not simply produce two equally valid facts. It produces an incompatibility requiring explanation. Perhaps the friend misremembered. Perhaps the notebook was altered. Perhaps the apparatus malfunctioned. Perhaps Wigner disturbed the system while opening the lab. The point is that once a common context exists, compatibility conditions apply. Contextuality does not abolish constraint.

A common world of agreement is therefore not given from nowhere. It is constructed through physical correlation. When observers communicate, they form a joint context that can support shared judgments. The more robust the network of agreement, the more classical and objective the fact appears. Scientific objectivity is an extension of this process. Instruments, observers, records, calibrations, replications, and publications form increasingly large pullback-like structures in which claims must agree across many routes of comparison.

In Wigner's friend, the simplest common world is the one formed when Wigner asks the friend what happened and receives an answer. But richer common worlds are possible. Wigner might inspect the apparatus, read the notebook, interview the friend, repeat the experiment, or compare environmental traces. Each additional comparison strengthens or complicates the pullback. If all routes agree, the shared fact becomes more stable. If some routes fail, the structure reveals an obstruction.

This view explains why public reality has a categorical flavor. Public facts are not merely contents broadcast to many observers. They are compatible sections over overlapping contexts. A fact is public when different observers, instruments, and records can be brought into a common refinement without contradiction. The pullback is the formal image of this shared domain.

Thus Wigner's friend does not force us to choose between private fact and absolute fact. It points to the construction of shared fact through common contexts of agreement. The friend's local outcome becomes Wigner's shared outcome by entering a pullback-like structure in which their records cohere over the laboratory event. Reality, at the classical level, is what survives this gluing.

7.3. Compatibility Conditions for Observer Records

The emergence of shared facts depends on compatibility conditions. A record does not become objective merely because it exists. It becomes objective when it can be related consistently to other records through admissible physical maps. The friend's memory, the apparatus pointer, the notebook entry, Wigner's later observation, and any subsequent report must fit together. If they do, they define a stable outcome across contexts. If they do not, the system contains a mismatch that must be resolved.

Compatibility has several layers. The first is internal compatibility within the friend's context. The friend's memory should agree with the apparatus reading. The notebook should agree with the friend's report. The local environment should not contain traces inconsistent with the recorded outcome unless some additional process intervened. This internal compatibility is what allows the friend to treat the outcome as locally definite.

The second layer is external compatibility within Wigner's context. Before opening the lab, Wigner's description must be compatible with his knowledge of the preparation, isolation, and dynamics of the laboratory. If he assigns a superposed state, that assignment should fit the assumed unitary evolution and sealed boundary conditions. If information has leaked from the lab, his context is different, and the state assignment must change accordingly.

The third layer is cross-context compatibility after communication. When Wigner opens the lab, the friend's report must be compared with Wigner's observation. The apparatus record must be compared with the friend's memory. Wigner's later description must be compared with the earlier external description through an update map. Compatibility does not require that every previous description literally match every later one. It requires that the transformations between them be intelligible and structure-preserving.

This point matters because Wigner's external superposition and the friend's internal record are not compatible by being identical. They are compatible if there exists an admissible transformation from the pre-communication structure to the post-communication structure that explains how Wigner becomes correlated with one record. The sealed-lab description belongs to one context. The shared-outcome description belongs to another. Compatibility is about lawful transition, not static sameness.

Observer records also have identity conditions. What makes a record the same record across context change? The answer cannot be mere microscopic sameness, because macroscopic records persist through enormous microscopic variation. A notebook page ages, a memory is biologically maintained, a digital file changes physical substrate, and a spoken report becomes another person's memory. The identity of a record is an equivalence class over many possible realizations. It is preserved when the outcome-relevant structure is preserved.

This is where equivalence-first reasoning becomes indispensable. The friend's memory of 0, the apparatus pointer at 0, the notebook mark for 0, and Wigner's later memory of hearing "0" are not identical physical objects. They are equivalent as records of the same outcome if the relevant maps connect them coherently. The shared fact is not one material token. It is an invariant across equivalent record tokens.

Compatibility conditions also distinguish genuine quantum contextuality from ordinary confusion. If records fail to glue because of noise, error, deception, or disturbance, the failure is mundane. If records fail to glue even under ideal conditions because no global assignment of classical values is possible, the failure is foundational. Wigner's friend is interesting because it gestures toward this deeper kind of obstruction: local facts may be meaningful without belonging to one universal classical truth table.

In categorical terms, compatibility asks whether a family of local sections can be assembled into a larger section over a common context. In HoTT terms, it asks whether the witnesses inhabiting local fact-types can be transported into a shared type without contradiction. In physical terms, it asks whether records can be compared, copied, and stabilized across observers. These are different languages for the same structural problem.

7.4. Classical Facts as Stabilized Relational Invariants

A classical fact is often imagined as something simply there. The pointer reads 0. The friend saw 1. The notebook contains a mark. The ordinary world encourages this realism because classical facts are usually stable, redundant, and publicly accessible. But quantum foundations suggests that classicality is an achievement. A classical fact is a relational invariant that has become stable across enough contexts to be treated as context-independent for practical purposes.

In Wigner's friend, the friend's local outcome becomes more classical as it becomes more robustly recorded. It is first correlated with the apparatus. Then with the friend's perception. Then with memory. Then perhaps with a notebook. Later with Wigner's observation. Later still with additional reports. At each stage, the same outcome-relevant structure is preserved across new physical carriers. The fact becomes classical by surviving translation across substrates.

This is why redundancy is central to classicality. A fragile microscopic state is easily disturbed and difficult to share. A macroscopic record is copied into many degrees of freedom. It can be

accessed in multiple ways. It can be checked, photographed, remembered, repeated, and communicated. Its identity is no longer tied to one delicate physical configuration. It becomes an equivalence class of mutually supporting traces.

The phrase stabilized relational invariant captures this. The fact is relational because it exists through correlations among systems. It is invariant because the same outcome structure persists across transformations, copies, and comparisons. It is stabilized because environmental, material, and social mechanisms preserve it against small disturbances. A classical fact is therefore not the opposite of relation. It is relation made robust.

This view also explains why classical facts appear observer-independent. Once a record has been widely stabilized, it becomes available through many routes. Any competent observer entering the relevant context can recover the same fact. The context-dependence has not vanished metaphysically; it has become practically invisible because the network of supporting contexts is so large and coherent. Objectivity is the limiting case of successful gluing.

Wigner's friend reveals the earlier stage before such stabilization is universal. The friend's record is local. Wigner has not yet joined the record network. The fact is real for the friend but not yet shared with Wigner. When communication occurs, the fact becomes stabilized across a wider relational structure. The difference between local quantum fact and classical public fact is therefore a difference in the scope and robustness of relational invariance.

This does not mean that reality is socially constructed in a shallow sense. Physical compatibility is not decided by agreement alone. Observers cannot vote an apparatus into having recorded 0 if all physical traces indicate 1. The relational invariant must be supported by the material structure of records. Communication extends physical correlation; it does not replace it with convention.

The same principle applies beyond Wigner's friend. Scientific facts are stabilized relational invariants across instruments, laboratories, observers, methods, and theoretical translations. A spectral line, a particle track, a genetic sequence, or a gravitational-wave signal becomes objective when its record structure survives many transformations. The quantum origin of facts does not make them weak. It explains why their strength comes from redundancy, compatibility, and invariant structure.

In the equivalence-first account, the classical world is not a separate ontological layer pasted onto quantum reality. It is the emergent domain of records that have become stable under the relevant equivalences and morphisms. The friend's fact becomes Wigner's fact when it survives the pullback into their shared context. More generally, a fact becomes public reality when it can be glued across contexts without losing its identity.

The conclusion of this section is therefore straightforward: communication is not an afterthought in quantum foundations. It is the operation by which local records become shared facts. Pullbacks represent the common worlds in which descriptions must agree. Compatibility conditions determine whether local records can be glued. Classical facts are the stabilized relational invariants that survive this process. Wigner's friend matters because it shows that objectivity is not the starting point of quantum reality; objectivity is what successful gluing produces.

8. HoTT and the Typing of Quantum Facts

Homotopy type theory offers a natural language for Wigner's friend because it does not force facts to appear as untyped, context-free assertions. In ordinary classical language, one is tempted to say either "the friend saw 0" or "the friend did not see 0," as if every proposition must already live inside a single universal space of truth. HoTT encourages a more disciplined formulation. A statement is made in a context. A proposition is represented as a type. A fact is represented by an inhabitant, a witness, or a term of that type. Identity is not mere syntactic equality but may involve paths, equivalences, transports, and higher structure.

This is exactly the kind of grammar needed for Wigner's friend. The friend's claim is not merely a bare sentence. It is a judgment made under the friend's context after a physical measurement has occurred. The friend has a witness: a memory, a perception, a record, an apparatus reading. Wigner's external state assignment is also not a bare sentence. It is a judgment made under Wigner's context before communication with the laboratory. The two judgments need not be collapsed into one type before a map between their contexts has been supplied.

HoTT helps prevent a common mistake in quantum foundations: erasing context and then treating the resulting contradiction as primitive. If we write the friend's fact and Wigner's state assignment as unindexed propositions in one global arena, they appear to collide. But if we write them as judgments in different contexts, the problem becomes more precise. We must ask whether a transport, comparison, refinement, or gluing operation exists that carries the relevant witness from one context to another. The issue is not simply whether a proposition is absolutely true or false. The issue is where it is typed, what witnesses it has, and how those witnesses behave under context change.

8.1. Propositions as Types and Facts as Inhabitants

In a HoTT-inspired reading, a proposition is not first treated as a detached Boolean value. It is treated as a type whose inhabitants count as evidence. To assert a proposition is to provide a term of the corresponding type. A fact is therefore not merely a label saying "true." It is an inhabited structure. This shift is especially useful for physical facts because physical facts are not

empty truth values. They are supported by records, traces, memories, apparatus states, environmental correlations, and admissible transformations.

For the friend inside the laboratory, the proposition “the friend saw outcome 0” corresponds to a type that may be inhabited in the friend’s post-measurement context. The inhabitant may be represented physically by the friend’s memory, the apparatus pointer, and the local record. The fact is not a floating abstraction. It is witnessed by the structure of the friend’s world after the measurement. If the friend saw 1 instead, then the type corresponding to “the friend saw outcome 1” would be inhabited in that context.

This differs from the classical habit of imagining that all propositions already have truth values independent of the context in which evidence exists. In HoTT language, the existence of an inhabitant is not context-free. A term is typed under assumptions, and those assumptions matter. The friend’s witness belongs to the friend’s context. The fact that the friend can produce a term of the type “I saw 0” does not automatically mean that Wigner, before opening the lab, has a term of the same type in his context.

This distinction is not subjectivism. It is typing discipline. A statement may be meaningful in one context because the required evidence exists there, while unavailable in another context because no transport map has carried the evidence there. The friend’s context contains the relevant witness. Wigner’s pre-communication context does not. Wigner may have a different type inhabited in his context, such as “the laboratory is assigned this superposed state” or “the lab will yield correlated friend-record outcomes if opened.”

This gives a more exact formulation of the apparent paradox. The friend has an inhabitant of an outcome-record type. Wigner has an inhabitant of an external-state-assignment type. These are not automatically contradictory because they are not terms of the same type in the same context. A contradiction would require a shared context in which incompatible inhabitants are asserted under the same typing rules.

Facts as inhabitants also clarifies why records matter. A record is a physical witness. It is what allows a proposition to become inhabited within a context. If the apparatus leaves no trace, if the friend has no memory, and if nothing in the laboratory is correlated with the outcome, then the relevant fact-type may not be inhabited in any usable way. Measurement becomes fact-producing because it produces witnesses.

From this perspective, the classical world is rich in inhabited types. The pointer reads 0, the notebook contains 0, Wigner heard the friend say 0, and the laboratory log stores 0. These are multiple witnesses that may inhabit equivalent or related fact-types. Objectivity emerges when witnesses can be transported among contexts and shown to cohere. A fact is strong when its type is inhabited redundantly across many compatible domains.

The slogan “propositions as types” therefore becomes a physical principle: a fact is not merely a truth value but a record-bearing possibility inhabited in a context. Wigner’s friend matters because it separates the existence of an inhabitant in the friend’s context from the availability of that inhabitant in Wigner’s context. The gap is not psychological. It is structural.

8.2. Friend-Facts as Contextual Witnesses

The friend’s fact is best understood as a contextual witness. After measurement, the friend does not merely believe that an outcome occurred. The friend is physically situated in a record-bearing context. The system, apparatus, perception, memory, and local environment form a chain of evidence. In HoTT terms, the friend’s context supports a term of an outcome type. The friend can produce a witness to the proposition “I saw 0” or “I saw 1.”

This witness is contextual because its validity depends on the physical assumptions and relations present inside the laboratory. The friend has interacted with the measured system. The apparatus has been read. The memory has formed. A local record exists. These conditions are not optional decorations. They are part of the context under which the judgment is typed. Remove the measurement interaction, erase the memory, or destroy the record, and the status of the witness changes.

A contextual witness is not inferior to an absolute fact. It is the form that a physical fact takes before being transported into wider agreement. The friend’s witness is real because it can guide action, support memory, and participate in later communication. If Wigner later enters the lab, the friend can report the result. The witness can be copied into Wigner’s context through speech, writing, or observation. It can become part of a larger compatibility structure.

However, before this transport occurs, Wigner does not possess the friend’s witness. Wigner may know that the friend will have seen some outcome, but he does not have the term corresponding to the friend’s actual result in his own context. This distinction between knowing that a type is inhabited and having a particular inhabitant is important. Wigner may know that the experiment produces a friend-record. But without communication, he lacks the specific record as a Wigner-accessible fact.

This gives a refined way to discuss the friend’s outcome. The friend’s fact is neither a global absolute automatically present in every context nor a mere illusion. It is an inhabited type in the friend’s context. Its extension beyond that context requires transport. Communication is the physical act that moves, copies, or relates the witness to another domain. Until then, the witness remains local.

The friend’s witness also has internal structure. It is not one indivisible atom. The friend’s report may depend on memory, visual perception, apparatus reliability, prior calibration, and linguistic encoding. Each of these can be represented as part of the witness structure. A strong witness is

one supported by multiple coherent sub-witnesses. A weak witness is one easily undermined by internal inconsistency. This matches ordinary experimental reasoning while giving it a precise type-theoretic form.

The possibility of error does not destroy the framework. If the friend misreads the apparatus, then the type “friend remembers 0” may be inhabited while the type “apparatus recorded 0” may not be. The mismatch reveals that different fact-types must be distinguished. Classical language often blurs them into “the result was 0.” HoTT encourages finer typing: result recorded by apparatus, result perceived by friend, result remembered by friend, result reported to Wigner, result logged publicly. These types are related but not identical.

This finer typing is valuable for quantum foundations because it prevents premature global simplification. Wigner’s friend is confusing partly because phrases like “the outcome” or “the fact” hide many levels of record. The friend’s contextual witness is one level. Wigner’s later witness is another. A public scientific record is another. The unity of the fact is achieved only when these witnesses are related by equivalences or compatible transport maps.

Thus the friend-fact is a contextual witness: an inhabited record-type inside the laboratory. It is not arbitrary, not unreal, and not automatically global. It is a local physical inhabitant awaiting possible gluing into broader objectivity.

8.3. Wigner-State Assignments as External Judgments

Wigner’s state assignment can also be expressed as a typed judgment. Before opening the lab, Wigner does not have the friend’s outcome witness. Instead, he has a different kind of evidence: the preparation procedure, the assumed isolation of the lab, the known measurement interaction, and the formal rules of quantum evolution. These support a judgment in Wigner’s context assigning a quantum state to the entire sealed laboratory.

This judgment is not the same type of fact as the friend’s record. The friend’s witness inhabits an outcome-specific type. Wigner’s judgment inhabits a state-assignment type. It says, roughly, that given Wigner’s context, the unopened lab is described by a certain quantum state over correlated friend-record alternatives. This is not a direct assertion that the friend saw no definite outcome. It is an external description of the laboratory as an object to which Wigner has not yet become internally correlated.

The difference between an outcome witness and a state-assignment judgment is essential. Many confusions in Wigner’s friend arise because these are treated as competing answers to the same question. But they are not the same type of judgment. The friend’s judgment is first-personally embedded in the post-measurement record context. Wigner’s judgment is externally assigned to the sealed laboratory. One is record-bearing from inside; the other is structure-describing from outside.

In HoTT-like discipline, Wigner's judgment must remain indexed by his context. It cannot be silently promoted into a universal statement overriding the friend's local witness. Nor can the friend's witness be silently promoted into Wigner's context before communication. Each judgment has its own domain. The work of interpretation lies in specifying the maps that relate the domains.

Wigner's external judgment may include a superposition of laboratory branches. Each branch contains a correlated friend-record structure. This means Wigner's state assignment can represent the possibility that the friend has a record without Wigner possessing that record as his own witness. The state assignment is about the whole lab. It is not a direct inhabitant of a type such as "Wigner has heard the friend say 0." That type remains uninhabited until Wigner interacts with the lab.

This clarifies the role of measurement from Wigner's standpoint. Wigner does not deny that the friend's measurement interaction occurred. He describes it as a unitary correlation within the sealed lab. The friend's memory and apparatus are included as physical subsystems. But because Wigner's context lacks a communication map from the friend's record to Wigner's record, the specific outcome is not available as a Wigner-fact.

The state-assignment judgment also carries assumptions. Wigner's description depends on believing the lab is isolated, the measurement interaction is known, and no decohering leakage has made the outcome effectively public. If those assumptions change, the judgment changes. Thus Wigner's state assignment is not an absolute proclamation. It is a term in a context defined by physical conditions. This is why the HoTT formulation is useful: the assumptions are part of the typing environment.

When Wigner opens the lab, the typing environment changes. Wigner now receives a witness of the friend's report. The previous external-state judgment must be transported, updated, or replaced by a new judgment in the joint context. This is not a contradiction. It is ordinary context extension. A term valid under one context may no longer be the appropriate term after new assumptions and witnesses are added.

The HoTT framing therefore allows Wigner's superposition to be taken seriously without making it tyrannical. It is a valid external judgment under the sealed-lab context. It is not a global annihilation of the friend's local fact. Its meaning is governed by the context in which it is typed and the maps available for later comparison.

8.4. Context Dependence Without Relativistic Confusion

A context-dependent account of quantum facts can easily be misunderstood as saying that truth is subjective, arbitrary, or dependent on personal opinion. HoTT helps avoid this confusion because context dependence is not looseness. It is discipline. A judgment is valid only under

specified assumptions. A term belongs to a type only under a context. Transport between contexts requires a map. Equivalence requires structure-preserving correspondence. Nothing in this framework licenses arbitrary contradiction.

The friend cannot simply choose any outcome. The friend's witness is constrained by the apparatus, memory, and laboratory record. Wigner cannot simply choose any state assignment. His judgment is constrained by the preparation, the isolation of the lab, and the rules of quantum dynamics. When Wigner and the friend later communicate, their records cannot remain freely independent. The joint context imposes compatibility conditions. Context dependence therefore increases precision rather than decreasing it.

The phrase "observer-relative" is also potentially misleading. In this framework, an observer is not a disembodied mind generating private reality. An observer is a physical system embedded in interactions. The relevant context is not mental preference but physical relation. The friend has a record because the friend interacted with the apparatus. Wigner lacks the friend's record because he has not yet interacted with the lab. The difference is relational and physical, not psychological.

This is why context dependence in quantum foundations is not the same as ordinary relativism. It is closer to coordinate dependence in geometry, but with stronger physical content. Different coordinate systems may describe the same geometric object differently, yet transformations relate the descriptions. Likewise, different quantum contexts may support different state assignments, yet admissible maps determine whether and how those assignments cohere. The reality lies not in one coordinate expression but in the invariant structure across transformations.

HoTT deepens this analogy by making identity itself structured. Two descriptions are not the same merely because one declares them the same. They must be connected by paths, equivalences, or transports. If such a connection exists, then the descriptions may represent the same physical content in different contexts. If no such connection exists, the failure is meaningful. It may reveal an obstruction, a measurement disturbance, an incompatible context, or a genuinely nonclassical feature.

Context dependence also preserves the difference between local and global truth. A type may be inhabited locally without producing a global section. A family of local witnesses may fail to glue into one universal witness. This is the type-theoretic analog of quantum contextuality. Local facts can be valid, yet the attempt to assemble them into a single classical truth table can fail. Wigner's friend dramatizes this at the level of observers and records.

This failure of global assembly does not mean that local facts are false. It means that the classical demand for one total fact-space is too strong. The friend's witness is valid in the

friend's context. Wigner's state-assignment judgment is valid in Wigner's context. A later shared witness may be valid in the joint context. The relations among these judgments must be tracked explicitly. Once they are tracked, the appearance of contradiction is replaced by a structured problem of transport and gluing.

The distinction is subtle but decisive. Relativistic confusion says, "Everyone has their own truth." Contextual discipline says, "Each judgment has a type, a context, and admissible transports; compatibility is required where contexts overlap." The first is philosophical looseness. The second is formal structure. Wigner's friend requires the second, not the first.

The conclusion of this section is that HoTT provides a grammar for quantum facts that ordinary classical language lacks. Propositions become types. Facts become inhabitants. Observer records become contextual witnesses. Wigner's state assignment becomes an external judgment under specified assumptions. Communication becomes context extension and transport. Objectivity becomes successful gluing of witnesses across contexts. This does not make reality subjective. It makes reality structurally typed. Wigner's friend is difficult because classical language erases the types. HoTT restores them.

9. Univalence and Physical Equivalence

Univalence gives this paper its deepest conceptual bridge between homotopy type theory and quantum foundations. In ordinary formal systems, identity is often treated as stricter than equivalence. Two objects may be equivalent, isomorphic, or structurally interchangeable, yet still counted as distinct because they are not literally the same symbol or construction. Univalence changes the foundational attitude. It says that equivalence can itself be treated as identity when the relevant structure is preserved. In mathematical practice, this captures what mathematicians often do informally: if two structures are equivalent in all the ways that matter, then they may be used as the same structure.

Quantum physics already lives close to this attitude. A raw state vector is not identical with a physical state if it contains representational features that make no observable difference. Multiplying a vector by a global phase changes the written expression but not the physical content. Changing gauge may alter the formal presentation while leaving observables invariant. Choosing a different basis may change coordinates without changing the underlying state. Different mathematical presentations can therefore represent the same physical reality. Physics is already equivalence-first in practice, even when its philosophical language remains state-first.

Wigner's friend pushes this equivalence-first principle beyond ordinary representational redundancy. The question is not only whether two mathematical expressions describe the same quantum state. The question is whether two context-indexed state assignments describe

compatible aspects of the same physical reality. The friend's definite record and Wigner's external superposition are not syntactically identical. They are not even the same kind of judgment. Yet they may be related by context change, communication, update, and gluing. The reality they describe is not found in either expression alone, but in the equivalence-governed structure connecting them.

Univalence therefore suggests a disciplined slogan: physical identity is not sameness of presentation; it is sameness under the transformations that preserve physical content. For quantum foundations, this means that the deepest object is not the isolated state vector but the equivalence class of admissible descriptions across contexts.

9.1. Equivalence as Identity in Foundations

The foundational force of univalence is that it elevates equivalence from a secondary relation to a principle of identity. In ordinary reasoning, one might say that two objects are distinct but equivalent. In univalent reasoning, if the equivalence preserves the relevant structure, then that equivalence can justify treating them as identical for the purposes of the theory. Identity is not reduced to visual sameness, syntactic sameness, or historical sameness. It is grounded in structure-preserving transformability.

This principle is powerful because physical theories rarely deal with naked things. They deal with representations, coordinates, models, fields, states, observables, transformations, symmetries, and invariants. The same physical situation may appear under many different descriptions. If each description were treated as a separate reality, physics would drown in redundancy. Instead, physics identifies descriptions that differ only by transformations with no physical effect. This is the practical physics analog of univalence.

In quantum theory, the physical object is already more abstract than its written expression. A state vector written in one basis may look very different from the same state written in another basis. A density operator may represent a subsystem after tracing out an environment. A wavefunction may be expressed in position space or momentum space. These presentations differ, but they can be connected by transformations that preserve the predictions of the theory. The physical content is not the ink on the page. It is the invariant structure across equivalent presentations.

Univalence gives a foundational language for this. If two descriptions are equivalent in the relevant sense, then they are not merely accidentally similar. They are different presentations of the same structure. This is precisely the move needed for an equivalence-first interpretation of quantum reality. We should not ask first what the uniquely real presentation is. We should ask which descriptions are equivalent under admissible physical transformations.

Applied to Wigner's friend, this means that the friend's and Wigner's descriptions should not be judged by syntactic sameness. The friend says, in effect, "I have a record of outcome 0." Wigner says, before communication, "I assign a superposed state to the unopened lab." These are not identical sentences. But the important question is whether they can be related within a broader structure without contradiction. If the friend's record can be transported into the joint Wigner-friend context, and Wigner's prior state assignment can be updated according to the physical interaction, then the two descriptions may be compatible components of one relational reality.

Equivalence as identity also prevents false metaphysical inflation. If two descriptions differ only because one includes an arbitrary convention, then the difference should not be reified. A global phase, a gauge choice, or a coordinate convention is not a secret additional layer of being. It is part of the presentation. The world is not bigger because our notation is redundant.

The same principle disciplines interpretations of observer-relative facts. If two observers later compare records and find compatible descriptions related by lawful maps, then their earlier context-specific accounts may be understood as different presentations within one coherent diagram. If no such equivalence or transport exists, then the failure is meaningful. The framework does not declare all viewpoints equivalent. It asks which ones are actually connected by structure-preserving relations.

Thus univalence supplies a standard for physical sameness: two descriptions are the same physical content when there exists an admissible equivalence preserving the relevant observables, records, transformations, and compatibility conditions. This is stronger than verbal agreement and weaker than syntactic identity. It is exactly the level at which quantum reality appears to operate.

9.2. Global Phase, Gauge, Basis, and Representation Equivalence

The simplest examples of physical equivalence occur before one reaches Wigner's friend. They arise in the ordinary grammar of quantum mechanics. A quantum state multiplied by a global phase is represented differently, but no measurement can detect that absolute phase by itself. The physical state is therefore not the vector alone but the ray: the equivalence class of vectors differing by nonzero scalar phase factors under the appropriate normalization. This is one of the clearest cases in which raw formal expression exceeds physical content.

Basis dependence provides another example. A state may be expanded in one basis or another. The coefficients change, the written expression changes, and the intuitive picture may change. Yet the underlying physical content is preserved when the transformation between bases is unitary and the observables are transformed consistently. To mistake a preferred basis of calculation for a preferred basis of being is to confuse representation with reality.

Gauge redundancy extends the same lesson. A gauge choice may simplify a calculation or make certain features visible, but gauge-dependent quantities are not directly physical unless they contribute to gauge-invariant observables. The mathematical description contains more structure than the physical situation requires. The real content is what survives gauge transformation. Again, the physical object is equivalence-governed, not presentation-bound.

Representation equivalence is broader still. The same physical system may be described by state vectors, density matrices, path integrals, operator algebras, tensor networks, categorical diagrams, or other formalisms. These descriptions may differ dramatically in appearance. Some emphasize dynamics. Some emphasize observables. Some emphasize entanglement. Some emphasize compositional structure. If they preserve the same physical predictions and transformation rules, they may be regarded as equivalent presentations of the same physical content.

These examples show that physics has long depended on a univalent habit: equivalent structure counts as the same content. This habit becomes explicit in an equivalence-first ontology. The world is not the coordinate chart. The world is not the gauge. The world is not the arbitrary phase. The world is what remains coherent under admissible transformations.

Wigner's friend adds a new layer. The relevant equivalences are no longer only internal to one observer's mathematical representation. They also concern the relation among context-indexed descriptions. The friend's local record and Wigner's external state assignment are not related by a simple change of basis. They are related, if at all, by a sequence of physical context changes: measurement, isolation, communication, and updating. The equivalence question becomes richer: can these descriptions be placed in one coherent diagram whose maps preserve the physically relevant records and predictions?

This is why the earlier examples matter. They train us not to overinterpret formal difference. If two state expressions differ by global phase, we do not infer two realities. If two gauge choices differ, we do not infer two worlds. If two bases differ, we do not infer two ontologies. Likewise, if the friend and Wigner assign different descriptions from different contexts, we should not immediately infer contradiction. We should first ask what kind of equivalence, transport, or gluing relation connects them.

However, not every difference is merely representational. Relative phase can matter when it affects interference. Gauge-invariant quantities matter. Basis choice can expose physically significant structure when a measurement context selects a basis. A difference between observer records can matter if it persists in a shared context. Equivalence-first reasoning does not flatten all distinctions. It distinguishes physical differences from artifacts of presentation.

The criterion is invariance under admissible maps. If a distinction survives the relevant transformations and can affect observable relations, it is physical. If it disappears under equivalence, it is not. This criterion will be essential in deciding when Wigner's and the friend's descriptions are compatible and when a genuine obstruction appears.

9.3. When Two State Assignments Describe the Same Reality

Two state assignments describe the same reality when they can be connected by transformations that preserve the relevant physical structure. This statement sounds simple, but Wigner's friend shows why it must be handled carefully. The friend's state assignment and Wigner's state assignment may not even assign states to the same object in the same way. The friend assigns an outcome-bearing description from inside the laboratory. Wigner assigns a quantum description to the whole unopened lab from outside. Their assignments differ in scope, context, and type.

Sameness of reality therefore cannot mean sameness of sentence or sameness of vector. It must mean compatibility within a structured diagram. The friend's record, Wigner's external description, and the later shared record describe the same reality if they can be placed into a network of maps where each transition is physically intelligible and the relevant invariants are preserved. The friend's record must be preserved through communication unless disturbed. Wigner's external assignment must update lawfully when interaction occurs. The later joint context must not contain irreducible contradictions among the records.

Consider the friend who observes outcome 0. Inside the lab, the friend has a record of 0. Wigner outside assigns a superposition over possible friend-record states. Later, Wigner opens the lab and hears the friend report 0. These descriptions can be made compatible if the superposed external state is understood as Wigner's pre-communication assignment, the friend's outcome is understood as a local witness, and the later report is understood as a communication-mediated extension of that witness into Wigner's context. The descriptions are not identical, but they are gluable.

By contrast, suppose the friend's internal context contains a stable record of 0, Wigner opens the lab without disturbing it, and all apparatus traces indicate 0, but Wigner's later record somehow claims that the friend saw 1 with no physical explanation. Then compatibility fails. The issue is not that observers are forbidden to disagree. They can disagree psychologically. The issue is that a coherent physical diagram must account for the disagreement. If no admissible morphism explains the mismatch, the family of descriptions does not describe one stable shared reality.

Thus equivalence-first quantum realism is not permissive relativism. It does not say that any state assignment is as real as any other. It says that state assignments are physically meaningful

to the extent that they participate in a coherent equivalence structure. The reality is the diagram, not the isolated assertion.

This has consequences for the meaning of objectivity. Objectivity does not require one observer-free description from which all others are merely shadows. It requires that descriptions from different contexts can be transported into common refinements without loss of the relevant structure. If friend, Wigner, apparatus, notebook, and later observers all support the same outcome under admissible comparison, then the outcome has become objective. Its objectivity lies in its invariance across context changes.

This also means that two apparently different states may be the same physical content when they differ only because their contexts differ. A reduced state for a subsystem and a pure entangled state for a larger system may both be legitimate descriptions when their relation is specified. A collapsed state for an observer with a record and an uncollapsed state for an external observer may both be legitimate if their domains and update maps are clear. The problem is not plurality of state assignments. The problem is untracked plurality.

The phrase “same reality” must therefore be used with mathematical care. There is sameness by gauge equivalence, sameness by unitary transformation, sameness by coarse-graining, sameness by record preservation, sameness by communication, and sameness by gluing into a common context. These are not all the same relation, but they share the same foundational pattern: identity is mediated by structure-preserving equivalence.

Wigner’s friend becomes less paradoxical when this pattern is honored. The friend’s reality and Wigner’s reality are not two disconnected worlds, nor are they forced into premature identity. They are context-indexed descriptions whose sameness or difference must be determined by the maps between them. Reality is what the diagram preserves.

9.4. Reality as an Equivalence Class of Compatible Descriptions

The final step is to state the equivalence-first ontology directly: reality is not a raw state vector but an equivalence class of compatible descriptions. This does not mean that reality is merely descriptive or linguistic. It means that our access to physical reality is through structures that admit multiple equivalent presentations, and the physical content is what remains invariant across them. A raw vector, a coordinate expression, a gauge-fixed field, a basis expansion, or a context-indexed state assignment is a presentation. The reality is the compatible structure those presentations jointly determine.

For Wigner’s friend, the relevant equivalence class includes the friend’s local record, Wigner’s external state assignment, the measurement morphism, the sealed-lab condition, the communication event, and the later shared record. No single item alone exhausts the physical reality of the scenario. The friend’s record captures local definiteness. Wigner’s external state

captures the quantum description of the unopened lab. The communication event captures the transition from local to shared fact. The compatibility of all these elements is the reality of the experiment.

This view changes the ontology of the quantum state. The state is not discarded. It is decentered. A quantum state remains an indispensable part of the physical description, but it is not the whole of physical being. It is one object in a structured network. Its meaning depends on context, equivalence, and admissible transformation. The deepest physical content lies in the invariant relations among states, records, observers, and operations.

Such an ontology is especially well suited to quantum mechanics because the theory repeatedly teaches us that raw distinctions are suspect. Global phase is not physical. Gauge artifacts are not physical. Unmeasured incompatible values do not form one classical table. Unknown states cannot be freely copied. Measurement outcomes become facts through record-producing interactions. Observer descriptions differ when interaction histories differ. These are not isolated oddities. They point toward an ontology in which equivalence and relation are primary.

Reality as an equivalence class of compatible descriptions also gives a precise account of why classical reality feels absolute. Classical facts are those that have become stable across a large equivalence class of records and contexts. They survive communication, copying, environmental redundancy, coarse-graining, and repeated access. Because they are so widely compatible, they appear context-free. But their apparent absoluteness is an achievement of stabilization, not a primitive metaphysical condition.

This approach also allows genuine nonclassicality to appear as failure of compatibility. If local descriptions cannot be glued into one global classical assignment, the obstruction is real. Quantum contextuality, Bell-type correlations, and Wigner's friend variants all reveal limits on the classical dream of a universal fact table. The equivalence-first view does not hide these failures. It makes them central. Where gluing succeeds, classical objectivity emerges. Where gluing fails, quantum structure announces itself.

The resulting picture is neither naïve realism nor subjective idealism. It is structural realism disciplined by equivalence. There is a real world, but it is not identical to any one privileged presentation. There are facts, but they are typed, contextual, and stabilized through relations. There are quantum states, but their physical content lies in what survives admissible transformations. There are observers, but they are physical systems embedded in the same diagram they help reveal.

Univalence provides the philosophical and mathematical permission for this move. If equivalent structures may be identified, then reality need not be tied to one syntactic form. Physical being can be understood as invariant structure across equivalent descriptions. Wigner's friend then

becomes a lesson in higher identity: the friend's record and Wigner's superposition are not the same at the level of raw expression, but they may belong to one higher equivalence class once context, transport, and gluing are included.

The conclusion is compact: quantum reality is not the state vector as written. It is the equivalence class of all compatible state assignments, records, transformations, and observer-context judgments. Wigner's friend is difficult because classical metaphysics looks for one absolute sentence. The equivalence-first view looks for the coherent diagram. In that diagram, identity is not imposed before relation. Identity is what relation, equivalence, and compatibility make possible.

10. Wigner's Friend as a Failed Global-Section Problem

The Wigner's friend scenario can now be stated in its sharpest categorical form: it is a failed global-section problem for classical facts. A global section would be a single assignment of definite facts that works across all relevant contexts at once. It would say, without qualification, which outcome occurred, what the friend saw, what Wigner should assign, what every possible observer may consistently claim, and how all possible measurements would resolve. This is the classical dream: one universal fact table, perhaps hidden from some observers but nevertheless complete.

Quantum theory resists this dream. Local contexts may support definite records, but these records do not always assemble into one observer-independent classical assignment. The friend's context supports a definite outcome. Wigner's pre-communication context supports a superposed description of the sealed laboratory. The later joint context supports a shared report. These local descriptions are not meaningless, but they do not behave as if they were all restrictions of one pre-existing global classical database.

The language of sheaves and global sections captures the issue with unusual precision. A presheaf assigns possible descriptions to contexts. A sheaf adds gluing conditions: compatible local descriptions can be uniquely assembled into a larger description. Quantum phenomena show that local consistency does not always imply global classical consistency. Wigner's friend dramatizes this failure at the level of observers and records. It asks whether the friend's definite fact, Wigner's external state, and all possible later comparisons can be understood as fragments of one classical whole. The answer is not simply yes. The global classical section is obstructed. What remains possible is relational gluing: local facts become shared where physical interaction supplies the maps needed for compatibility.

10.1. Local Facts and the Temptation of Global Completion

A local fact is a fact supported within a particular physical context. The friend's record is local in this sense. It is not local because it is weak, private, or unreal. It is local because it is physically grounded inside the friend-laboratory context. The friend has seen the apparatus, formed a memory, and perhaps written the outcome down. The fact is supported by a real chain of correlations. Within that context, the statement "the friend saw 0" or "the friend saw 1" may be fully meaningful and operationally decisive.

The temptation of global completion is to take every such local fact and place it into one universal ledger. If the friend saw 0, then the global ledger says the friend saw 0. If Wigner does not know this, Wigner is simply ignorant. If another observer has not yet arrived, that observer is also ignorant. The fact itself is imagined to be complete, absolute, and already present in the same form for all possible contexts. This is the classical picture.

But quantum mechanics complicates this move. Wigner's external state assignment is not merely the statement "I do not know which definite outcome occurred." Under the idealized conditions of the thought experiment, Wigner may treat the entire unopened lab as a quantum system. The lab is not simply a box containing one hidden classical entry. It may be described as a superposition of correlated friend-record alternatives. This description is not a local friend-fact, but it is also not arbitrary. It is licensed by Wigner's physical relation to the sealed laboratory.

The classical urge is to complete the story by insisting that one of the friend-record alternatives must already be selected in the same global sense for everyone. This may be a legitimate move within some interpretations, but it is not forced by the bare structure of the scenario. It imports a classical global-section assumption: every local fact must be the restriction of one global fact. Wigner's friend shows why that assumption requires defense.

A local fact can be definite without being globally completed in the classical sense. The friend's outcome is definite for the friend because the friend occupies the record-bearing context. Wigner's pre-communication context does not yet contain that record as an accessible fact. The later joint context may contain it after communication. The fact's range of availability grows through physical interaction. It is not automatically available everywhere merely because it is locally real somewhere.

This distinction is subtle but essential. To deny automatic global completion is not to deny the friend's record. It is to deny that the friend's record must immediately function as an entry in a universal classical table for all possible observers and all possible descriptions. The record is local before it is shared. It becomes shared through morphisms. It becomes objective through compatibility. It becomes classical through stabilization.

The temptation of global completion also appears when one asks what “really happened” before Wigner opened the lab. Classical grammar wants one answer. Quantum structure allows a more careful response: in the friend’s context, a record was formed; in Wigner’s context, the sealed lab was assigned a superposed state; in the later joint context, a shared outcome may be established. These statements are not evasions. They are context-indexed judgments. The demand for one unindexed sentence is precisely the classical demand under examination.

A global classical section would erase these distinctions. It would assign one definite outcome across every context in advance. But the Wigner’s friend setup suggests that such erasure may conceal the very structure quantum theory is trying to show us. The more faithful account begins with local facts and asks how far they can be extended. Some extend smoothly. Some require update. Some expose obstruction. The global section is not assumed; it is tested.

10.2. Quantum Contextuality and Obstructions to Gluing

Quantum contextuality is the broader principle that the value assigned to a measurement outcome cannot always be regarded as independent of the measurement context. In classical physics, one imagines that properties exist with definite values whether or not they are measured and regardless of which compatible set of measurements is chosen. Quantum theory undermines this picture. It suggests that the conditions under which a question is asked are part of what makes an answer meaningful.

This is precisely the sheaf-theoretic idea of obstruction. One may have locally consistent assignments that cannot be glued into a single globally consistent assignment. Each local patch works. Each local measurement context may admit coherent outcomes. But when one tries to assemble all patches into one universal table, contradictions appear. The failure is not in any one local patch. It is in the assumption that all local patches must be restrictions of one global classical object.

Wigner’s friend can be read as a contextuality problem extended to observers. The friend’s measurement context contains a definite outcome. Wigner’s external context contains a different kind of description. The later joint context contains a communication-mediated record. Each context has its own internal logic. The question is whether all these contexts can be glued into one global classical account in which every observer’s fact, every possible measurement, and every state assignment coexist as pre-existing entries.

The obstruction appears because the friend’s definite record and Wigner’s external superposition are not simply two entries in the same classical table. They are different context-indexed structures. To glue them classically, one must usually demote one of them. Either Wigner’s superposition becomes mere ignorance about an already absolute outcome, or the friend’s definite record becomes merely branch-relative, relational, or otherwise not globally

absolute. Each interpretation chooses a way to manage the obstruction. The obstruction itself is the structural fact that the naïve global table does not arise automatically.

Gluing requires compatibility on overlaps. If two local descriptions overlap in a common context, their shared content must agree. Before Wigner opens the lab, the friend and Wigner do not share the outcome record. Their overlap is limited to preparation, laboratory setup, and the expectation that a measurement interaction occurred. After Wigner opens the lab, the overlap expands. The friend's report and Wigner's observation must now agree if no disturbance or error has intervened. The gluing condition strengthens as the physical connection strengthens.

This is why contextuality does not imply contradiction. Local descriptions are not required to agree where no overlap exists. They are required to agree where the diagram supplies a common refinement. The friend's fact and Wigner's external state are not contradictory merely because they differ. They would become contradictory only if the framework demanded that they occupy the same context with incompatible content. The categorical approach avoids this forced collision by tracking the maps.

The obstruction to gluing is also not always total. Wigner's friend does not imply that no shared facts can ever be formed. On the contrary, shared facts form constantly. Wigner can open the lab, hear the friend's report, inspect the apparatus, and enter a joint context. What fails is not all gluing. What fails is the assumption of one prior global classical section that existed independently of these gluing operations. Relational gluing succeeds locally and progressively; global classical completion fails as a primitive assumption.

This gives a balanced view. Quantum contextuality is not a license for unlimited fragmentation. It is a constraint on how local facts may be assembled. Some assemblies are possible. Some are not. The nonclassical character of quantum reality lies in the pattern of possible and impossible gluings. Wigner's friend is a vivid observer-level example of this pattern.

10.3. Why Classical Global Sections Fail

A classical global section fails because it assumes more definiteness than quantum structure permits. It assumes that all relevant propositions can be assigned context-independent truth values at once. It assumes that every measurement has a pre-existing result, every observer's record can be placed into one universal ledger, and every state assignment is ultimately reducible to ignorance about that ledger. Wigner's friend shows that these assumptions cannot be made innocently.

The first reason for failure is incompatibility. Quantum observables do not generally admit simultaneous sharp values. A state may support definite prediction for one observable and only probabilistic prediction for another. A classical global section attempts to assign values to all of

them anyway. This creates a hidden table that exceeds the operational structure of the theory. The result is not merely unknown; it is not a valid global object under the rules of quantum mechanics.

The second reason is contextuality. The meaning of an outcome depends on the measurement arrangement and the relations among system, apparatus, observer, and environment. A value cannot always be detached from the context that makes it measurable. A classical global section detaches all values from all contexts. It treats them as if they were already present independently. Quantum theory resists this detachment.

The third reason is observer embedding. Wigner's friend makes the observer part of the quantum system. The friend is not outside the formalism. The friend's measurement, memory, and record may be included in Wigner's external description. Once this is allowed, the distinction between fact-holder and object-described becomes context-dependent. What is background for the friend is object for Wigner. A classical global section assumes that these roles can be flattened without loss. They cannot.

The fourth reason is the physical nature of communication. Facts become shared through interactions. The friend's record becomes available to Wigner only when some physical channel connects them. A classical global section treats shareability as automatic. Quantum theory treats it as mediated. The difference matters. Before communication, Wigner does not simply lack a database password. He lacks the physical correlation that would make the friend's outcome a Wigner-accessible fact.

The fifth reason is representational redundancy. Even if one invokes a universal quantum state, the raw formal object contains nonphysical distinctions: phase conventions, basis choices, gauge choices, and other redundancies. A classical global section confuses a complete physical description with a complete classical inventory. But physical completeness in quantum theory does not mean that every classical proposition has a definite value. It means that the structure gives the correct amplitudes, correlations, and transformation rules for possible contexts.

The failure of classical global sections is therefore not a failure of realism. It is a failure of an overly classical form of realism. The world may be real without being representable as a single context-free table of definite facts. The real structure may be presheaf-like, relational, typed, and equivalence-governed. In such a world, local facts exist, but their extension to larger domains depends on gluing conditions.

Wigner's friend makes this vivid because all the usual classical stabilizers are challenged. A human observer has a record, yet another observer can, in principle, assign a superposed state to the entire record-bearing lab. The ordinary intuition that a human observation must

immediately settle reality for all possible contexts becomes questionable. The friend's fact is real locally. Wigner's description is real externally. The global classical table is what fails.

This failure does not prevent later objectivity. When Wigner opens the lab, a shared fact may be established. But that shared fact arises through context refinement, not because it was already present as an untyped global entry. The classical world is recovered as a stabilized domain of successful gluing. It is not assumed at the foundation.

The deepest lesson is that quantum reality is not deficient because it lacks a classical global section. Rather, the demand for such a section was the error. Quantum mechanics describes a world in which compatibility, not preassigned totality, is primary. Facts are local first, shared later, and global only when the structure permits.

10.4. Relational Gluing as the Repair of Apparent Contradiction

If the classical global section fails, what repairs the apparent contradiction? The answer is relational gluing. Local descriptions become parts of a broader reality when physical maps connect them and compatibility conditions are satisfied. The friend's record, Wigner's external state assignment, and the later shared report are not forced into one timeless sentence. They are organized into a diagram whose morphisms show how each description relates to the others.

Relational gluing begins with respect for locality of context. The friend's fact belongs to the friend's post-measurement context. Wigner's superposed state belongs to Wigner's pre-communication context. The joint outcome belongs to the post-communication context. Once these domains are distinguished, the contradiction no longer appears as an immediate logical inconsistency. It appears as a question of whether the domains can be connected coherently.

The repair occurs when Wigner interacts with the lab. The friend's local witness is transported into Wigner's context through communication. Wigner's prior state assignment is updated through his new correlation with the laboratory. The joint context now contains a shared record. If the friend's report, apparatus trace, and Wigner's observation agree, the local descriptions glue into a broader fact. The fact becomes more objective because it is now invariant across more contexts.

This gluing is relational because it depends on actual physical links. It is not enough to declare that two descriptions refer to the same event. They must be connected by admissible operations: measurement, memory, communication, comparison, preservation, and transformation. The friend's record must be physically carried into Wigner's context. Wigner's state assignment must be lawfully transformed by the interaction. The shared fact must survive the maps that relate the contexts.

Relational gluing also preserves the asymmetry between before and after communication. Before Wigner opens the lab, the friend's outcome is not yet Wigner's record. After Wigner opens the lab, it may be. This is not a change from unreality to reality. It is a change in scope. The fact moves from local definiteness to shared definiteness. The repaired account avoids both extremes: it does not deny the friend's outcome, and it does not force Wigner to possess it before interaction.

In this sense, relational gluing is the constructive replacement for the failed global section. Instead of assuming one universal classical fact table from the start, the theory builds objectivity by joining contexts. Where contexts overlap and compatible witnesses exist, facts can be glued. Where no admissible gluing exists, the failure is informative. It marks a boundary of classical representation.

The repair is also compatible with multiple interpretations. A collapse theorist may say the gluing reveals an outcome made objective by collapse. A many-worlds theorist may say gluing occurs branch-relatively. A relational quantum theorist may say facts are always relative to systems and become shared through interaction. A QBist may interpret the update as agent-centered expectation revision. The categorical account does not require that all these interpretations become identical. It identifies the structural feature they must all address: the relation between local records and broader compatibility.

Relational gluing also explains why science works in a quantum world. Scientific objectivity is not the existence of facts from nowhere. It is the construction of stable records across contexts. Instruments measure, records persist, observers communicate, laboratories replicate, and theories translate. Each successful connection extends the domain over which a fact is invariant. Classical knowledge is therefore a large-scale gluing achievement.

Wigner's friend is a minimal drama of this process. One observer has a fact. Another observer has a state assignment. A boundary prevents immediate sharing. A later interaction permits comparison. The apparent contradiction is repaired not by flattening the story, but by respecting its diagrammatic structure. The world is not one classical sheet laid over all contexts. It is a sheaf-like structure in which local sections may or may not glue.

The conclusion of this section is that Wigner's friend should be read as a failure of global classical completion and a success of relational reconstruction. The global section fails because quantum facts are not preassembled into one observer-free truth table. Relational gluing repairs the situation by showing how local records become shared through physical interaction. This is the categorical meaning of quantum objectivity: not absolute fact from nowhere, but compatibility achieved through morphisms.

11. Interpretational Consequences

The categorical and HoTT framing does not eliminate the familiar interpretations of quantum mechanics. Instead, it reorganizes the questions they must answer. Wigner's friend has always served as a stress test for interpretation because it asks whether measurement creates an objective fact, whether all outcomes persist in branching structure, whether quantum states are personal probability assignments, whether facts are relative to systems, or whether some new physical process interrupts unitary evolution. The equivalence-first account does not simply pick one of these views and discard the others. It asks what each interpretation does with contexts, morphisms, records, witnesses, gluing, and equivalence.

This shift is useful because many disputes in quantum foundations are framed as if the central issue were the status of one state vector. Is the state real? Does it collapse? Does it branch? Is it knowledge? Is it relational? The present paper suggests that the deeper issue is not the state alone but the structure of state assignment. A quantum state has meaning within a context. Measurement produces record-bearing contexts. Communication builds shared contexts. Classical facts emerge when local witnesses glue across a network of compatible records. Interpretations differ in how they explain this structure, but none can avoid it.

Wigner's friend therefore becomes a diagnostic device. Collapse interpretations must explain when and why a local record becomes an objective fact. Many-worlds must explain why branch-relative records appear definite and how branch-local gluing produces ordinary objectivity. QBism must explain how agent-indexed expectations coordinate across communication without collapsing into solipsism. Relational quantum mechanics must explain how facts relative to systems become shared without requiring a view from nowhere. The equivalence-first synthesis asks whether these approaches can be understood as different projections of a broader categorical structure.

11.1. Collapse Interpretations and Objective Fact Creation

Collapse interpretations respond to Wigner's friend by asserting that measurement does something physically decisive. At some point, the quantum state no longer evolves as a mere superposition of alternatives. One outcome becomes actual. The friend's measurement is not simply an internal correlation described differently by Wigner; it is a fact-creating event. On this view, Wigner's external superposition is either not physically correct after the friend's measurement or is only a temporary calculational device reflecting Wigner's ignorance of the objective result.

The strength of collapse interpretations is that they preserve the ordinary seriousness of records. When the friend sees outcome 0, outcome 0 has occurred. The friend's memory, the apparatus, the notebook, and the environment are aligned around a real event. Wigner may not

know the result, but the result exists independently of Wigner's later interaction. This matches much of common experimental language. Laboratories do not usually treat measurement outcomes as merely relative until someone outside the room checks them. They treat records as facts once they are reliably produced.

From the categorical perspective, collapse interpretations give a strong answer to the gluing problem. They say that the local friend-record is not merely local in a weak sense. It is the emergence of an objective section. Once collapse occurs, the global classical assignment includes the friend's outcome. Wigner's later interaction reveals or confirms a fact already made objective by the measurement. The gluing problem is solved by adding a physical process that selects one local section as globally actual.

This move has costs. Collapse interpretations must specify what counts as a collapse-triggering event. Is it interaction with a macroscopic apparatus, irreversible amplification, consciousness, gravity, spontaneous localization, environmental entanglement, or something else? If collapse is a genuine physical process, then it should have conditions, scales, dynamics, and perhaps observable consequences. Without such specification, collapse risks becoming a name for the transition one wanted to explain.

Wigner's friend sharpens the demand. If the friend's measurement collapses the state, then Wigner cannot consistently assign a coherent superposition to the entire laboratory after that collapse, except as ignorance. But if Wigner could in principle perform an interference experiment on the sealed lab, collapse would forbid or modify the expected result. The interpretation must therefore say whether Wigner's external superposition is physically impossible, practically impossible, or merely epistemically inappropriate.

In the equivalence-first language, collapse interpretations privilege fact creation over contextual plurality. They insist that at some point the diagram of possible state assignments is pruned into a single outcome-bearing structure. This gives a clear path to objectivity but risks imposing global classicality too early. The question is whether the collapse mechanism is independently justified or merely introduced to protect the classical global section.

Nevertheless, collapse views contribute an important insight. Records matter. Measurement is not just another line in a formal calculation. The formation of robust, irreversible, outcome-bearing structure is central to the emergence of classical reality. Even if one rejects primitive collapse, any adequate theory must explain why record-bearing contexts behave as if collapse occurred for the systems embedded in them.

The categorical account can therefore absorb part of the collapse intuition without accepting collapse as primitive. It can say that measurement creates a record-bearing context in which effective collapse is valid. The friend's context has projected onto an outcome. Whether that

projection is also an objective universal physical event is a further interpretive claim. Collapse interpretations answer yes. The equivalence-first account treats that yes as one possible way of globalizing local record formation.

11.2. Many-Worlds and Branch-Relative Records

Many-worlds interpretations respond differently. They preserve universal unitary evolution and deny that a physical collapse selects one outcome globally. In the Wigner's friend setup, the friend-lab system evolves into a superposition of branches. In one branch, the friend sees outcome 0. In another, the friend sees outcome 1. Each branch contains a definite record relative to the branch. Wigner's external superposition is therefore not ignorance about one actual result. It is a description of the branching structure that includes multiple internally definite friend-records.

The strength of many-worlds is that it takes the universality of quantum mechanics seriously. The friend, apparatus, memory, and laboratory are all physical systems. They evolve according to the same quantum rules as anything else. No special collapse boundary is needed. Wigner's external description is not demoted. It is the larger structure within which the friend's definite experiences appear as branch-relative records.

From the categorical perspective, many-worlds replaces a single global classical section with a branching global quantum structure. The friend's fact is not absolute across the whole state; it is definite within a branch. Wigner's state assignment includes the branch structure. When Wigner opens the lab, Wigner becomes correlated with one branch-relative friend-record. The shared fact emerges within that branch. Objectivity is not global uniqueness of outcome; it is stable agreement among observers within the same branch.

This fits well with the idea that classical facts are stabilized relational invariants. In many-worlds, each branch contains its own network of stabilized records. The friend's memory, apparatus, notebook, and Wigner's later observation agree within a branch. Decoherence prevents practical interference between macroscopically distinct branches, making branch-relative facts appear classical and robust. Public reality is therefore branch-local gluing.

The cost is that many-worlds must explain why probability and actuality feel the way they do. If all outcomes occur in different branches, what does it mean to expect one result rather than another? Why should amplitudes guide rational expectation? What exactly makes a branch a branch rather than an approximate emergent decomposition? How should one count or compare branches when the branching structure may be continuous, approximate, and basis-sensitive? These are not minor issues. They are central to making the interpretation physically and philosophically coherent.

Wigner's friend sharpens the issue of perspective. For the friend, one outcome is experienced. For Wigner, before interaction, the whole lab may remain in a superposition. Many-worlds can accommodate both by saying that the friend's definiteness is branch-relative while Wigner's superposition is the broader branch-containing description. But the interpretation must still explain why the friend should not regard the other branch as part of the same experienced reality. The answer usually appeals to decoherence, record isolation, and the inability of branches to communicate after effective separation.

In equivalence-first language, many-worlds treats the raw universal quantum structure as more fundamental than any single branch's classical facts, but it still relies on equivalence and gluing within branches. A branch is not merely a term in a chosen basis. It must be a stable record-bearing structure preserved under decoherence and coarse-graining. Thus many-worlds also depends on a relational account of classical fact formation. It does not escape the need for context; it relocates context inside branch structure.

The equivalence-first account can learn from many-worlds that Wigner's external superposition should not be dismissed too quickly. The external quantum description may contain real structure not reducible to ignorance. At the same time, the equivalence-first account resists treating the universal wavefunction as a simple absolute database. Even in many-worlds, physical content must be extracted through branch-relative records, decoherence-defined contexts, and equivalence classes of stable macroscopic structure.

Thus many-worlds offers a powerful solution to Wigner's friend by allowing both Wigner's superposition and the friend's definite record to be real at different structural levels. Its central contribution to the present framework is the idea that definiteness may be local to a stable component of a larger quantum structure, rather than globally unique from the start.

11.3. QBism and Agent-Indexed State Assignment

QBism gives an even more radical answer: the quantum state is not an objective physical thing assigned from nowhere. It is an agent's personal probability assignment for that agent's possible future experiences. On this view, the friend and Wigner may legitimately assign different quantum states because they are different agents facing different possible experiences. The friend has experienced a measurement outcome. Wigner has not. Their state assignments differ because their situations differ.

The strength of QBism is that it refuses to reify the quantum state as a cosmic object. It treats state assignment as action-guiding expectation. Measurement is not the passive revelation of a pre-existing property; it is an agent's intervention into the world and the subsequent experience that updates the agent's expectations. In Wigner's friend, the friend updates after seeing the

result. Wigner updates only after interacting with the lab. There is no paradox in their assigning different states because quantum states are not observer-independent inventories.

This fits naturally with the context-indexed language of the present paper. A state assignment belongs to a context. It is not made from nowhere. The friend's context contains an outcome witness. Wigner's context does not. A QBist reading treats this as fundamental: each agent assigns states relative to that agent's own experiences and expectations. The quantum formalism constrains rational betting behavior, but it does not describe an objective wavefunction floating above all agents.

From the categorical perspective, QBism emphasizes the fibers over agents. Each agent-context has its own state assignments. Communication between agents changes their expectations and creates new correlations among their records. Wigner opening the lab is not the discovery of a state that was already binding on him; it is an experience that requires him to update. The friend's prior experience is real for the friend, but it does not automatically become Wigner's experience before interaction.

The cost of QBism is that it must explain the objectivity of the world without making the quantum state objective. If all state assignments are personal, why do agents converge? Why do records agree? Why does scientific practice produce stable public facts? QBism typically answers that the external world is real but that quantum states represent agents' expectations about interactions with it, not the world's intrinsic condition. The world pushes back through experience. Agents cannot choose outcomes freely. Their probability assignments are personal, but experience is not under personal control.

Wigner's friend sharpens this issue because the friend's record seems more than personal expectation. It is embodied in apparatus, memory, notebook, and environment. A purely agent-centered language can risk underdescribing the physical record structure that makes shared facts possible. The friend's experience is not just a private sensation; it is correlated with material systems that Wigner may later inspect. Any QBist-compatible account must therefore explain how personal experiences become embedded in public record networks.

In equivalence-first language, QBism is valuable because it prevents premature absolutization of the state vector. It teaches that state assignment is not identical with reality. However, the equivalence-first account adds more explicit structure around records and gluing. It asks not only how agents update expectations, but how physical witnesses are transported across contexts. Objectivity arises not merely from agents revising beliefs, but from the stable compatibility of records under physical maps.

QBism also gives a helpful warning against the phrase "what the quantum state encodes." If the state is an agent's assignment, then it does not encode all facts about the world. It encodes

expectations for an agent's possible experiences. The world itself is not reducible to the state. This aligns with the paper's claim that quantum states are not classical databases. They are structured tools for navigating possible interactions.

The equivalence-first synthesis can therefore incorporate a QBist insight while refusing a fully private framing. Quantum state assignments are agent- or context-indexed, but the physical world includes record-producing interactions that constrain how those assignments can later be glued. The friend and Wigner may assign different states, but when they communicate, their records must enter a compatibility structure. That structure is not merely personal. It is physical.

11.4. Relational Quantum Mechanics and System-Relative Facts

Relational quantum mechanics offers perhaps the closest existing interpretation to the categorical framing developed here. It holds that facts are relative to physical systems. A property is not simply possessed absolutely; it is actual relative to another system with which interaction has occurred. In Wigner's friend, the friend's outcome is a fact relative to the friend because the friend interacted with the measured system. Before Wigner interacts with the lab, that outcome is not yet a fact relative to Wigner. When Wigner later interacts with the friend and laboratory, a fact becomes established relative to Wigner as well.

This interpretation directly matches the idea of context-indexed judgments. The friend's statement is valid in the friend-system relation. Wigner's external state assignment is valid in the Wigner-lab relation before communication. The later shared record is valid in the Wigner-friend relation after interaction. Facts are not arbitrary; they are indexed by physical interaction. The relational view therefore dissolves the paradox by refusing the assumption that every fact must be absolute from the outset.

From the categorical perspective, relational quantum mechanics can be understood as assigning facts to morphisms or interaction-defined contexts rather than to systems in isolation. A fact arises when systems interact. The friend's measurement is a morphism producing a friend-relative record. Wigner's later communication with the friend is another morphism producing a Wigner-relative record. The structure of reality is the network of these relations.

The strength of relational quantum mechanics is that it treats observers and systems symmetrically. A human friend is not metaphysically special. Any physical system may serve as a reference system relative to which facts are actual. This removes the need for consciousness-based collapse and avoids privileging Wigner's external view. The friend's fact is real relative to the friend. Wigner's description is real relative to Wigner. Later interaction relates them.

The cost is that relational quantum mechanics must explain how a stable shared world emerges from system-relative facts. If every fact is relative, how do we recover the apparent objectivity of ordinary reality? The answer must involve consistency conditions among interactions. When

systems interact and exchange records, their relative facts must become compatible in the appropriate contexts. Objectivity is not primitive but emergent from a dense network of mutually consistent relations.

This is where category theory strengthens the relational view. It supplies a formal language for objects, morphisms, pullbacks, compatibility, and gluing. Relational quantum mechanics says facts are relative to systems. The categorical account asks how those relative facts compose. Which diagrams commute? Which local records glue? Which transformations preserve record identity? Which attempted global sections fail? These questions sharpen relational ontology into structural mathematics.

HoTT also strengthens the relational view by typing facts. The friend's fact is an inhabited type in the friend's context. Wigner's external assignment is a different judgment in Wigner's context. A later shared fact is a witness in an extended context. Relational quantum mechanics supplies the physical intuition; HoTT supplies the grammar of context-dependent validity.

The equivalence-first approach can be seen as a categorical extension of relational quantum mechanics. It agrees that facts are not absolute classical entries from nowhere. It agrees that interaction is central. It agrees that Wigner's friend is resolved by tracking whose fact is relative to which physical relation. But it adds that physical sameness must be handled through equivalence, and that shared reality is produced through gluing of compatible state assignments and records.

This addition matters because relational accounts can otherwise sound too permissive. If facts are simply relative, one may worry that incompatible realities proliferate. The categorical account prevents this by imposing compatibility conditions wherever morphisms connect contexts. If Wigner and the friend communicate, their records must cohere unless a physical disturbance explains the mismatch. Relativity of fact does not mean absence of constraint.

Relational quantum mechanics therefore fits naturally within the paper's thesis. It provides the ontological intuition that facts are interaction-relative. The equivalence-first categorical framework provides the structural account of how those facts become shared, when they fail to globalize, and why classical objectivity emerges as stabilized compatibility rather than primitive absoluteness.

11.5. Equivalence-First Physics as a Categorical Synthesis

The equivalence-first view developed in this paper is not simply another interpretation placed beside collapse, many-worlds, QBism, and relational quantum mechanics. It is better understood as a structural synthesis that asks what all viable interpretations must account for. Any interpretation of Wigner's friend must explain context-indexed state assignment, record

formation, communication, compatibility, and the emergence or failure of global facts. The equivalence-first framework makes these requirements explicit.

From collapse interpretations, it accepts the seriousness of record formation. Measurement produces something physically consequential. The friend's record cannot be dismissed. A record-bearing context is not just a formal branch in an equation; it is the basis for local action, memory, and later communication. However, the equivalence-first view does not immediately treat collapse as a universal primitive. It first treats measurement as a morphism into a record-bearing context.

From many-worlds, it accepts the legitimacy of Wigner's external quantum description. The sealed laboratory may be described as a larger quantum system, and the friend's record may be embedded inside that description. However, the equivalence-first view does not treat the raw universal wavefunction as a simple classical database. It insists that physical content is extracted through branch structure, decoherence, record equivalence, and context-relative gluing.

From QBism, it accepts that state assignment is not view-from-nowhere description. Quantum states are tied to the situation of the assigning agent or context. Wigner and the friend may assign different states without contradiction. However, the equivalence-first view places greater emphasis on physical records, witnesses, and compatibility maps. It does not reduce shared reality to personal updating.

From relational quantum mechanics, it accepts that facts are relative to systems and interactions. The friend's outcome is a fact relative to the friend before it is a fact relative to Wigner. However, the equivalence-first view adds a categorical account of composition. It asks how relative facts glue into shared facts and when attempts at global classical completion fail.

The synthesis is therefore not a lowest-common-denominator compromise. It is a change of level. Instead of beginning with the question "Which interpretation is correct?" it begins with the diagrammatic structure that every interpretation must represent. There are contexts. There are state assignments over contexts. There are morphisms corresponding to measurement, isolation, communication, restriction, and update. There are witnesses corresponding to records. There are equivalences corresponding to sameness of physical content. There are gluing conditions corresponding to objectivity. There are obstructions corresponding to nonclassicality.

This categorical structure allows the quantum state to be decentered without being discarded. The quantum state remains essential, but its meaning lies in its place within a network. A state assignment is not an absolute monolith. It is a local section over a context. Its physical content is determined by the transformations that preserve it, the records it predicts or encodes, and the ways it can be glued to other assignments.

The equivalence-first account also gives a disciplined answer to the question of what cannot be encoded in quantum states. A quantum state cannot encode physical distinctions that vanish under equivalence. It cannot encode unlimited retrievable classical information. It cannot encode simultaneous definite answers to all incompatible questions. It cannot encode a context-free classical truth table for every observer. It cannot by itself encode semantic meaning without a system of interpretation. What it can encode are amplitudes, correlations, transition structures, and relational possibilities whose physical meaning emerges through context and interaction.

Wigner's friend becomes the test case for this answer. The friend's definite record is not absent from reality. Wigner's superposition is not simply nonsense. The universal classical fact table is what fails. The repair is not to choose one sentence and discard the rest, but to construct the diagram in which each sentence has its proper type and context. Reality is the coherent equivalence class of compatible descriptions, not the isolated dominance of one description.

This synthesis also clarifies the status of objectivity. Objectivity is not denied. It is reconstructed. A fact becomes objective when it is stable across contexts under admissible maps. The friend's outcome becomes shared with Wigner through communication. Scientific facts become public through replication, instrumentation, calibration, publication, and durable records. The classical world is the large-scale domain in which gluing succeeds so reliably that context-dependence becomes practically invisible.

The deepest interpretational consequence is that quantum foundations should move from state ontology to relational consistency. The state alone is not enough. Nor is the observer alone. Nor is the outcome alone. The fundamental structure is the coherent diagram of contexts, morphisms, records, and equivalences. This is why category theory and HoTT are not decorative additions. They provide the natural grammar for a world in which identity is equivalence, facts are typed, and objectivity is gluing.

Equivalence-first physics therefore reframes Wigner's friend as a lesson rather than a paradox. The lesson is that the world is not assembled from absolute facts first and relations second. It is assembled from relations that stabilize facts. Quantum states participate in this assembly, but they do not exhaust it. The physical real is what survives transport across contexts, what remains invariant under equivalence, and what can be glued into shared structure without contradiction.

12. Conclusion: Reality as a Coherent Diagram of Equivalences

Wigner's friend begins as a paradox about measurement, but it ends as a challenge to the metaphysics of description. The thought experiment asks whether quantum reality can be represented as a single observer-independent inventory of completed facts. The friend inside the laboratory has a definite record. Wigner outside the sealed laboratory may assign a superposed state to the entire lab. If both descriptions are forced into one untyped classical arena, they appear to contradict one another. But if they are treated as context-indexed judgments related by physical morphisms, the contradiction becomes a more precise structural question: how do local records, state assignments, observer contexts, and later communications glue into shared reality?

The answer developed in this paper is equivalence-first. A quantum state is not a classical database. A measurement is not merely the retrieval of a hidden entry. A record is not automatically global. An observer is not outside the system. A fact is not a disembodied Boolean value. Reality is better understood as a coherent diagram of contexts, morphisms, witnesses, transformations, and equivalences. The friend's outcome is real as a local record. Wigner's superposition is real as an external state assignment under the sealed-lab context. Their shared fact emerges when communication creates a joint context in which records can be compared and stabilized.

This conclusion does not trivialize the measurement problem. It sharpens it. The central question is no longer whether the friend or Wigner has the one privileged description. The question is what structure makes their descriptions mutually intelligible. Category theory provides the language of contexts and morphisms. HoTT provides the language of typed facts, witnesses, transport, and equivalence. Univalence supplies the foundational insight that equivalent descriptions may count as the same physical content when the relevant structure is preserved. Wigner's friend is therefore not merely a puzzle about observers. It is an invitation to replace state-first ontology with relational consistency.

12.1. The End of the Quantum State as a Classical Container

The first conclusion is that the quantum state should not be treated as a classical container of all facts. It is too structured, too contextual, too constrained, and too equivalence-governed for that role. A classical container stores entries that can, in principle, be read, copied, and compared. A quantum state does not behave this way. Measurement yields a specific outcome in a chosen context, not a complete extraction of the state. Unknown states cannot be copied at will. Incompatible observables cannot all be assigned simultaneous definite values. Formal distinctions such as global phase or gauge choice do not necessarily correspond to physical distinctions.

This does not make the quantum state unreal or unimportant. It makes it nonclassical. A quantum state encodes amplitudes, correlations, transition structures, and possible relations among systems. It governs what may happen under specified interactions. It can describe entanglement across systems and predict the statistics of future measurements. But it is not an observer-free spreadsheet in which every outcome for every possible observer is already written as a classical fact.

Wigner's friend exposes this limit dramatically. If the quantum state were a classical container, then either the friend's outcome would have to be stored as a single global entry, or Wigner's external superposition would have to be the single global entry. But neither move preserves the full structure of the scenario without interpretive cost. The friend's context contains a definite record. Wigner's context may contain a superposed description of the unopened lab. The state does not function as a simple container resolving this plurality. It functions as part of a context-indexed structure.

The end of the quantum state as a classical container is therefore not the end of realism. It is the end of a naïve picture of realism. The physical world need not be a database in order to be real. It may instead be a network of constrained relations in which facts arise through interaction and become objective through compatibility. The quantum state participates in this network, but it is not identical to the whole network.

This shift also clarifies what cannot be encoded in quantum states. A state cannot encode nonphysical distinctions that vanish under equivalence. It cannot encode unlimited retrievable classical information. It cannot encode a full table of definite answers for all incompatible questions. It cannot encode observer-independent semantic meaning without an interpretive and operational context. Most importantly for Wigner's friend, it cannot encode a universal classical fact table that every observer must already share before the physical relations that make sharing possible.

Once this is accepted, the quantum state becomes less paradoxical and more precise. It is not a failed classical object. It is a successful quantum object. The error lies in asking it to perform a classical metaphysical task it was never suited to perform.

12.2. Measurement as Record-Producing Relation

The second conclusion is that measurement should be understood first as a record-producing relation. Whatever deeper interpretation one prefers, measurement changes the physical structure of a context. It correlates a system with an apparatus. It correlates the apparatus with an observer. It may correlate the observer with a memory, a notebook, an environmental trace, or a public archive. A measurement is not simply the passive reading of a pre-existing value. It is a morphism that creates and stabilizes outcome-bearing structure.

For the friend, this record-producing relation is decisive. The friend's context after measurement differs from the friend's context before measurement. Before measurement, there are possible outcomes. After measurement, there is a record. The friend's memory and apparatus are aligned around one outcome. The local world has acquired a witness. In HoTT terms, the relevant outcome type has become inhabited in the friend's context.

For Wigner, before communication, that same record-producing relation may be described differently. The friend's record is part of the sealed laboratory system. Wigner may assign a superposed state to the entire lab, including the friend-record alternatives. The friend treats the record as contextual background. Wigner treats the record as object-level structure inside a larger quantum description. This is not contradiction by itself. It is a difference in categorical placement.

Measurement as relation also explains why collapse appears differently across interpretations. For collapse theories, record formation may correspond to an objective physical selection. For many-worlds, record formation occurs branch-relatively within a unitary global state. For QBism, measurement is an agent's action and subsequent experience. For relational quantum mechanics, measurement establishes a fact relative to an interacting system. The equivalence-first account identifies the common structural core: a measurement creates a new context in which certain records become available and certain judgments become admissible.

The friend's record is therefore neither a mere illusion nor an automatically universal fact. It is a real local witness produced by physical interaction. Its extension beyond the laboratory requires further relations. Wigner must open the lab, receive a signal, inspect the apparatus, or otherwise become correlated with the record. Shared fact is not produced by abstract declaration. It is produced by physical communication.

This gives a more grounded view of objectivity. A fact is not objective because it is imagined from nowhere. It becomes objective when records survive transport across contexts. Measurement begins the process by producing a local record. Communication expands it. Redundancy stabilizes it. Agreement across admissible maps makes it public. The classical world is the domain in which such record-producing relations have become so dense and reliable that facts appear simply given.

12.3. Observer Facts as Gluable Local Judgments

The third conclusion is that observer facts are gluable local judgments. The friend's statement "I saw outcome 0" is not merely a private mental event, nor is it automatically a universal fact available to all observers. It is a judgment made in a context that contains the physical witness required for that judgment. The friend's fact is local because it is grounded in a particular

interaction history. It is gluable because it can be transported into wider contexts through communication and comparison.

This formulation avoids two common errors. The first error is absolutism: assuming that every local fact must immediately be an entry in a global classical truth table. The second error is subjectivism: assuming that context-indexed facts are merely personal opinions or disconnected private realities. The friend's fact is neither. It is a physically grounded local judgment subject to later compatibility conditions.

When Wigner opens the lab, a new joint context forms. The friend's memory, the apparatus reading, Wigner's perception, and any written record may now be compared. If they agree, the local judgment glues into a broader shared judgment. If they disagree, the discrepancy demands a physical explanation. Contextuality does not permit arbitrary contradiction. It specifies where compatibility must be enforced.

This gluing process is the real source of shared objectivity. A fact becomes stronger as it survives more transports. The friend remembers the outcome. The notebook records it. Wigner hears it. The apparatus confirms it. Later observers may read the report. Each successful preservation expands the equivalence class of compatible witnesses. The fact becomes classical by becoming stable across a network.

HoTT makes this structure clear. A fact is an inhabited type in a context. Communication is transport or context extension. Shared objectivity is the successful gluing of witnesses into a larger type. Failed gluing indicates either ordinary error or deeper nonclassical obstruction. This grammar is better suited to Wigner's friend than untyped classical language, because it preserves the difference between local validity and global availability.

The idea of gluable local judgments also explains why scientific facts can be both human-accessible and physically constrained. Scientific objectivity is not the elimination of observers. It is the disciplined coordination of observer-indexed records through instruments, methods, calibrations, replications, and public communication. Wigner's friend is a miniature version of this larger structure. It shows that facts become public by being physically related, not by being metaphysically imposed from outside all contexts.

Thus the friend's fact should be taken seriously, but not flattened. It is a local witness capable of gluing. Wigner's external assignment should be taken seriously, but not absolutized. It is a context-indexed judgment awaiting update upon interaction. Their coherence is found not in a single sentence but in a diagram.

12.4. From State Ontology to Relational Consistency

The final conclusion is that quantum foundations should move from state ontology to relational consistency. State ontology asks what the state is and whether it is real, epistemic, collapsed, branching, objective, or subjective. These are important questions, but they become misleading if the state is isolated from the relational structure in which it is assigned and used. Wigner's friend shows that the meaning of a state depends on context, and the meaning of a fact depends on record-producing interaction.

Relational consistency asks a different set of questions. What contexts are present? What records exist in each context? What morphisms connect the contexts? Which descriptions can be transported? Which witnesses can be glued? Which distinctions survive equivalence? Which local facts fail to extend into a global classical section? These questions do not replace quantum mechanics. They reorganize its foundations around the structure that quantum mechanics repeatedly displays.

In this view, reality is not made of isolated states first and relations second. Reality is the coherent diagram of relations through which states, records, and facts acquire physical meaning. A quantum state is one kind of object in this diagram. A measurement is a morphism. A record is a witness. A communication event is a gluing operation. An equivalence is an identity of physical content. A classical fact is a stabilized relational invariant. A quantum obstruction is a failure of global classical assembly.

This relational-consistency approach also gives a new meaning to the phrase "basis of reality." If quantum states are foundational, they are not foundational as containers of all classical information. They are foundational as generators of relational possibility. They structure what can be measured, how systems can become correlated, how records can form, and how observers can enter shared contexts. But the physical real is not exhausted by the state as written. It includes the admissible transformations and equivalences that determine what the state means.

Wigner's friend therefore teaches humility about description. No observer is granted a view from nowhere. The friend's internal record is real, but local. Wigner's external state is valid, but contextual. The later shared fact is objective, but constructed through interaction. The universal classical truth table is not the foundation. It is, at best, an emergent approximation where gluing succeeds broadly enough to make context-dependence invisible.

The paper's central thesis can now be stated in its final form: quantum reality is not a raw state vector, not a private experience, not a classical database, and not a set of absolute facts waiting outside all contexts. Quantum reality is the equivalence-governed coherence of state assignments, records, witnesses, measurements, and communications across physically related

contexts. Wigner's friend is difficult because it exposes the failure of classical global completion. It is illuminating because it points toward the correct replacement: relational gluing under equivalence.

The conclusion is therefore not that reality dissolves into perspectives. It is that reality is the structure that makes perspectives compatible where compatibility is physically possible and reveals obstruction where classical compatibility fails. The world is not less real because it is relational. It is more precisely real: real as invariant, real as gluable, real as witnessed, real as transported, real as coherent under equivalence.

13. References

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